Temple 8024 **Numerical Differential Equations II** **Spring 2017**

Problem Set 4

(Out Thu 03/09/2017, Due Thu 03/30/2017)

Problem 6

(1) Download the code `temple8024_weno_claw.m` from the course website and run it. Plot the numerical approximation obtained with 80 cells, together with the true solution.

(2) Change the code so that it approximates the linear advection equation with smooth initial conditions. Perform a numerical error analysis and report the scheme's convergence rate. Explain why we do not obtain fifth order, even though a fifth order WENO reconstruction is used.

Problem 7

Write an at least third order accurate WENO code[1] that solves the 2D advection equation

$$\phi_t + u\phi_x + v\phi_y = 0\,, \quad (x,y) \in]0,1[^2, t \in]0,T[$$

with the velocity field $(u,v) = (-\psi_y, \psi_x)$ where $\psi(x,y,t) = \cos(\pi t/T)\sin^2(\pi x)\sin^2(\pi y)$, with $T = 4$.

Run your code on the initial conditions $\phi(x,y,0) = \sqrt{(x-0.25)^2 + (y-0.3)^2} - 0.1$, and plot the zero contour $\Gamma(t) = \{(x,y) : \phi(x,y,t) = 0\}$ at times $t \in \{0,1,2,3,4\}$. Do so for three resolutions: one for which the results look bad, one for which they look descent, and one for which they look very good.

Problem 8

The fire control of New South Wales would like to test a new approach to impede the propagation of bush fires: In a checkerboard pattern, regular squares of 1 km × 1 km are sprayed so that the propagation speed of the fire front is slowed down.

(1) Write a program that simulates the advance of a fire front that starts in the center of an untreated square and moves outward in its normal direction, with a velocity that is 1 km/h in the untreated squares, and ε km/h in the sprayed squares.

(2) Create a function $d(\varepsilon)$, where d denotes the largest distance of the fire from the origin at the final time $T_{\text{final}} = 24$ h. Do so by running your simulation for a whole range of values $\varepsilon \in [\frac{1}{100}, 1]$. Also plot the shape of the burning region for $\varepsilon \in \{\frac{1}{100}, \frac{1}{3}, \frac{4}{5}, 1\}$.

(3) Explain your results: Are there any critical values of ε at which a transition in the fire shape occurs? Is the idea of a checkerboard spraying a good one?

[1] Use the code `temple8024_weno_claw.m` for inspiration.