Temple 8023 **Numerical Differential Equations I** **Fall 2016**

Problem Set 6

(Out Wed 11/16/2016, Due Wed 11/30/2016)

Problem 12

Consider the advection-reaction equation

$$u_t + u_x = r(u) + s(x, u)$$

on $x \in [0, 2\pi[$ with periodic boundary conditions, and zero initial conditions $u(x, 0) = 0$. The solution $u(x, t)$ represents a chemical concentration ($0 \leq u \leq 1$), which is advected with constant velocity, and modified by a bistable reaction term $r(u) = u(1-u)(u-\frac{1}{2})$ and a localized source term $s(x, u) = a \exp\left(-10(x-\pi)^2\right)(1-u)$, where $a > 0$ is a parameter.

(1) Write a program that approximates the true solution with sufficient accuracy, and run the simulation on the two cases $a = 0.5$ and $a = 1$. Plot both solutions at times $t \in \{2, 8, 40\}$. Explain your observations.

(2) There is a critical threshold value a_c, such that for $a < a_c$, the solution behaves like the case $a = 0.5$, and for $a > a_c$, the solution behaves like the case $a = 1$. Find a_c numerically, up to at least 0.1% accuracy. Remember that your scheme's global truncation error must be sufficiently small.

Note: This problem set is only half of the prior problem sets.