Temple 8023 **Numerical Differential Equations I** **Fall 2016**

Problem Set 5

(Out Wed 11/02/2016, Due Wed 11/16/2016)

Problem 10

Consider the steady-state convection-diffusion equation

$$-\varepsilon u_{xx} + u_x = 1$$

in $]-1,1[$ with $u(-1) = 0 = u(1)$. For each choice of $\varepsilon \in \{10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}\}$, write a program that uses not more than 500 grid points, and approximates the solution with acceptable accuracy. Visualize your computational grids, and plot your numerical approximation, and its difference to the true solution.

Problem 11

We would like to investigate whether it is better to install a radiator under a window or at the opposite wall. Consider the evolution of temperature $u(x,y,t)$ be described by the heat equation

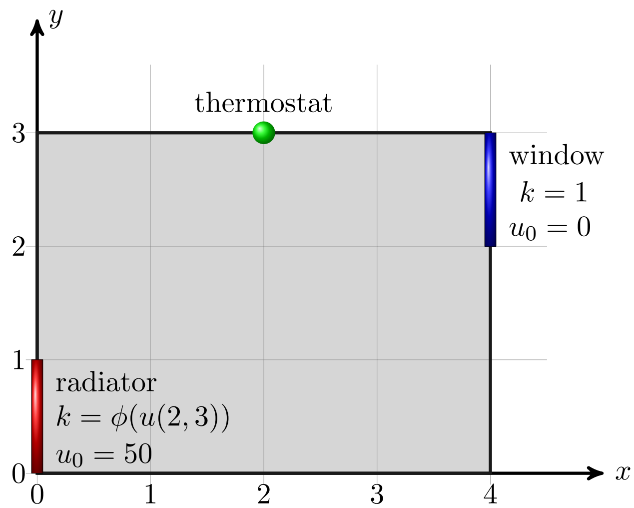
$$\begin{cases} u_t = \nabla^2 u & \text{in } \Omega \\ \frac{\partial u}{\partial n} = k(u_0 - u) & \text{on } \partial\Omega \\ u(x,y,0) = 0 \end{cases}$$

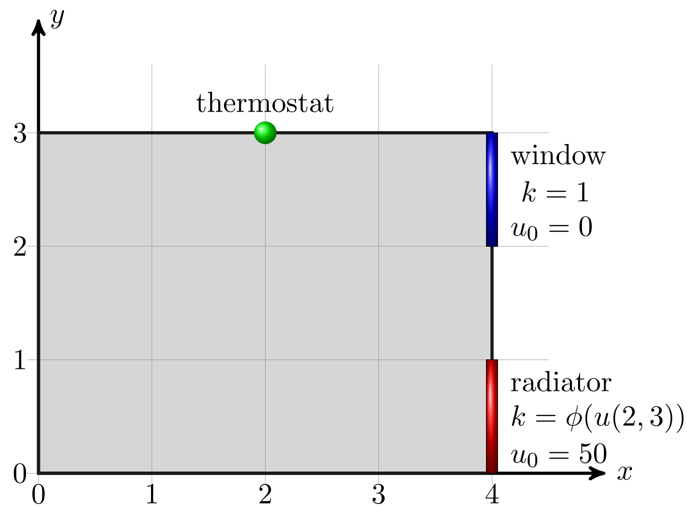
in a 2D room $\Omega =]0,4[\times]0,3[$, where

- $u_0 = 0$ and $k = 1$ at a window $\{4\} \times [2,3]$,
- $u_0 = 50$ and $k = \phi(u(2,3))$ at the radiator $\{0\} \times [0,1]$ or $\{4\} \times [0,1]$, and
- $k = 0$ (perfect insulation) everywhere else at the boundary.

The radiator's heating properties depend on the temperature at a thermostat via

$$\phi(u) = \begin{cases} 10 & u \leq 20 \\ 0 & u > 20\ . \end{cases}$$





(1) Write a program that approximates the true solution with sufficient accuracy in space and time, and simulate both rooms.

(2) In both rooms, plot the time evolution of the temperature at $P = (1, 2)$ and $Q = (3, 2)$. Choose the final time large enough that you see a time-periodic behavior towards the end.

(3) What is the average temperature in each room (averaged over space and over one periodic cycle)?

(4) Which room is more comfortable to be in, and why?

(5) Which design is more energy efficient (the loss of energy is the integral over $\frac{\partial u}{\partial n}$ over the window)?