Temple 8023 **Numerical Differential Equations I** **Fall 2016**

Problem Set 3

(Out Wed 10/05/2016, Due Wed 10/19/2016)

Problem 6

Consider the r-step BDF method of the form

$$\sum_{j=0}^{r} \alpha_j U^{n+j} = kf(U^{n+r})\ . \tag{1}$$

(1) Write a short Matlab program that for each r automatically computes the vector of coefficients $\vec{\alpha} = (\alpha_0, \dots, \alpha_r)$, such that (1) is globally r-th order accurate.

(2) Plot the regions of absolute stability for the BDF methods $r \in \{0, 1, \dots, 8\}$.

(3) What can you say about zero-stability?

(4) Do the regions of absolute stability grow or shrink with increasing order?

Problem 7

Consider the linear ODE system

$$\begin{cases} \vec{u}'(t) = A \cdot \vec{u}(t) \\ \vec{u}(0) = \mathring{\vec{u}} \end{cases} \tag{2}$$

where

$$A = \begin{pmatrix} -5000 & 4999 & 0 & 0 \\ 4999 & -5000 & 0 & 0 \\ 0 & 0 & 0 & -10 \\ 0 & 0 & 10 & 0 \end{pmatrix} \quad \text{and} \quad \mathring{\vec{u}} = \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix} .$$

We would like to approximate the solution of (2) at $\vec{u}(1)$, using an ODE solver with equidistant time steps. We are happy to be within 5% accuracy.

(1) Calculate the true solution $\vec{u}(1)$.

(2) Find numerically the maximum time step that yields the desired accuracy, using ...

  (a) forward Euler

  (b) backward Euler

  (c) RK4

  (d) BDF2

  (e) BDF6[1]

(3) Explain your observations.

(4) Plot the approximate solution obtained by backward Euler (with maximum admissible time step), together with the true solution.

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[1] Here you are allowed to cheat and use the correct solution values for the first $k-1$ time steps.