Temple 8023 **Numerical Differential Equations I** **Fall 2016**

Problem Set 2

(Out Wed 09/21/2016, Due Wed 10/05/2016)

Problem 4

The van der Pol oscillator

$$u'' - \mu(1 - u^2)u' + u = 0 \tag{1}$$

is a linear oscillator (case $\mu = 0$), augmented by a nonlinear damping[1] term.

(1) Someone claims that for any $\mu > 0$, equation (1) has a stable limit cycle solution. Verify this numerically, and plot the limit cycles (in the u–u'-plane) for $\mu \in \{\frac{1}{2}, 1, 2, 4, 8\}$.

(2) For any $\mu \geq 0$, the periodic solution has a given period T. Perform a sequence of numerical runs for various values of μ, and plot the function $T(\mu)$, for $\mu \in [0, 50]$.

(3) Contest: Who gets the largest value of μ, for which you find $T(\mu)$ numerically?

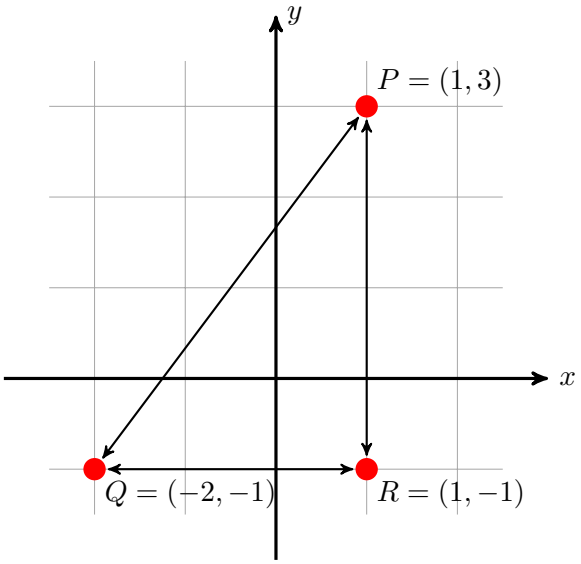
Problem 5

A supernatural creator[2] places three planets P, Q, R with masses 3, 4, 5, respectively, at the positions shown in the figure to the right. At time $t = 0$, the planets are at rest. The laws of motion are given by pairwise gravitational attraction $F = \frac{m_1 m_2}{d^2}$.

(1) Set up a first order system of ODE that describes the dynamics.

(2) Compute the solution of the arising system of ODE up to time $T = 75$, and plot the trajectories of the three planets in the xy-plane.

(3) Explain your observations.



Some remarks:

1. This problem is computationally challenging. You may need a high order ODE solver with adaptive time stepping to get close to the true solution.
2. You can check whether you are close to the true solution by diving the time step in half and comparing the results.

[1] It is "damping" in a generalized sense, as it can reduce as well as increase the energy of the system.

[2] Insert your favorite example.