Temple 8023 **Numerical Differential Equations I** **Fall 2016**

Problem Set 1

(Out Wed 09/07/2016, Due Wed 09/21/2016)

Problem 1

The convection-diffusion equation

$$u_t + u_x = \varepsilon u_{xx} + f(x) \tag{1}$$

describes the evolution of a solvent concentration u in a channel flow of velocity 1, while solvent is inserted at a rate $f(x)$. Solutions of (1) converge (in time) to the steady state ODE

$$-\varepsilon u_{xx} + u_x = f(x)\ . \tag{2}$$

Consider (2) in $]-1, 1[$ with $f(x) = 1$, and zero inflow $u(-1) = 0$. We are interested in two cases:

(A) small diffusion: $\varepsilon \ll 1$, where $u(1) = 0$, and

(B) no diffusion: $\varepsilon = 0$.

Find the solutions to (A) and (B). Plot the sequence of solutions to (A) for $\varepsilon \in \{10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}\}$. Explain your observations.

Problem 2

The telegraph equation $u_{tt} + 2du_t = u_{xx}$ describes the evolution of a signal in an electrical transmission line (`en.wikipedia.org/wiki/Telegraph_equation`). Consider $x \in [-\pi, \pi)$ with periodic boundary conditions. Find the solution by a Fourier approach. Show that the waves e^{ikx} travel with frequency dependent velocities, while being damped with time.

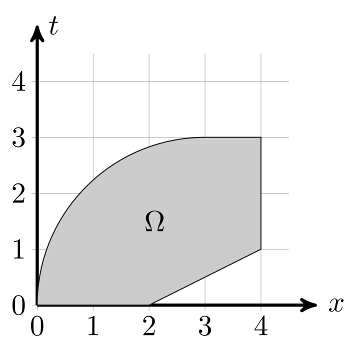
Problem 3

Given below are four PDE on a domain Ω. In each example:

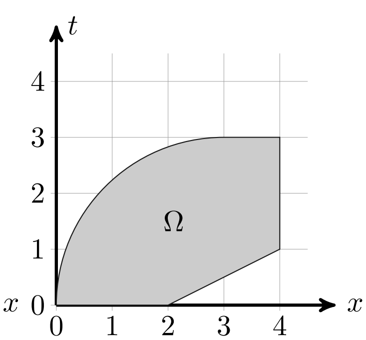
(1) Provide boundary and initial conditions, such that the problem is well posed.

(2) Explain/prove why your construction leads to a well posed problem.

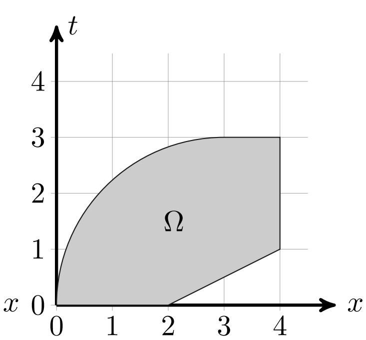
(3) Describe which physical problem is modeled, in particular enlighten the physical meaning of boundary conditions.



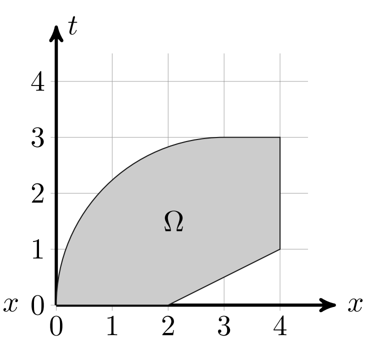
$u_t + u_x = 0$



$u_t + u_{xx} = 0$



$u_{tt} + u_{xx} = 0$



$u_{tt} - u_{xx} = 0$