Temple 9200 **Computational Methods for Flow Problems** **Spring 2015**

Problem Set 7 (in collaboration with Scott Ladenheim)

(Out Fri 04/10/2015, Due Wed 04/22/2015)

On this problem set we are solving the 2D Stokes equations

$$\begin{cases} -\nabla^2 \mathbf{u} + \nabla p = \mathbf{f} & \text{in } \Omega = (0,1)^2 , \\ \nabla \cdot \mathbf{u} = 0 & \text{in } \Omega = (0,1)^2 , \\ \mathbf{u} = \mathbf{g} & \text{on } \partial\Omega, \end{cases}$$

using `deal.ii` . This software package, as well as extensive documentation, are available for download from `http://www.dealii.org` . We recommend that you use the `deal.ii` virtual machine which is a compiled, working version and contains installations of the text editor emacs and the visualization softwares VisIt (`https://wci.llnl.gov/simulation/computer-codes/visit`), and Paraview (`http://www.paraview.org`). This virtual machine can be downloaded from `http://www.math.clemson.edu/~heister/dealvm/` . Please do not hesitate to contact Scott Ladenheim with any questions you may have pertaining to issues using `deal.ii` on this problem set.

Problem 9

Download the file `temple9200_lid_driven_cavity.tar` from the course website `http://math.temple.edu/~seibold/teaching/2015_9200/` . Unpack this file using the command `tar -xvf temple9200_lid_driven_cavity.tar` in the examples sub-directory of the `deal.II` directory. This will set up a directory of the same name with the following required files, `CMakeLists.txt` and `temple9200_stokes_dealii_lid_driven_cavity.cc` . The boundary conditions for the lid-driven cavity problem are zero-velocity boundary conditions on the lateral and bottom sides of the domain, and a horizontal velocity, $\mathbf{u} = (1,0)^T$, across the top of the domain.

a) Run the code, plot the solution (using, e.g., Paraview, VisIt) and describe what you observe. Is the inf-sup condition satisfied? Correct the code and send a copy of the correct version with the label `yourfamilyname_stokes_dealii_lid_driven_cavity.cc` to `saladenh@temple.edu` .

b) In the current implementation of the code, the Stokes system is solved using a direct method. Read the documentation about solvers in `https://www.dealii.org/8.2.0/doxygen/deal.II/group_Solvers.html` and modify the code so that the linear system is solved with an appropriate Krylov subspace method, for instance MINRES or GMRES. Set the residual stopping criterion to 10^{-10}. Produce a table that lists the number of iterations it takes to converge as the mesh is refined. List the number of degrees of freedom at each refinement level in your table as well. Explain in which sense, and why, the performance of the iterative solver deteriorates as the mesh is refined.

c) Download and run the file `mit18086_navierstokes.m` from the coarse website. Describe and explain the differences between the solutions that the two codes produce.

Problem 10

Download the file `temple9200_manufactured_solution.tar` from the course website `http://math.temple.edu/~seibold/teaching/2015_9200/` . Unpack this file using the command `tar -xvf temple9200_manufactured_solution.tar` in the examples sub-directory of the `deal.II` directory. This will set up a directory of the same name with the following required files, `CMakeLists.txt` and `temple9200_stokes_dealii_manufactured_solution.cc` .

a) For this problem we have manufactured a right-hand side function $\mathbf{f}$ so that it produces to the following Stokes solution:

$$\begin{cases} \mathbf{u}(x,y) = (\sin(\pi y), \sin(\pi x))^T, \\ p(x,y) \;= x + y - 1. \end{cases}$$

Run the code and plot the solution. When the code finishes running, a table showing the L^2 velocity and pressure errors at each refinement level is given. Determine the orders of convergence from these values and produce a log-log plot of the decrease in error as the mesh is refined. Do these orders make sense, considering the types of elements used?

b) Modify the code so that you manufacture a right hand side that corresponds to the following solution

$$\begin{cases} \mathbf{u}(x,y) = (\sin(\pi y), \sin(\pi x))^T, \\ p(x,y) \;= \sin(\pi(x+y)). \end{cases}$$

What are the orders of convergence for this problem. Why do we obtain a different behavior?