Temple 9200 **Computational Methods for Flow Problems** **Spring 2015**

Problem Set 5

(Out Wed 03/11/2015, Due Wed 03/25/2015)

Problem 5

Consider the Lighthill-Whitham-Richards (LWR) model for traffic flow $\rho_t + f(\rho)_x = 0$, where $f(\rho) = \rho(1-\rho)$ is the Greenshields flux, and initial conditions

$$\rho(x,0) = \begin{cases} 0.3 & x \le 0 \\ 0.6 & 0 < x \le 1 \\ 0.9 \quad \text{for} & 1 < x \le 2 \\ 0.6 & 2 < x \le 3 \\ 0.4 & 3 < x \, . \end{cases}$$

a) Use the software *particleclaw*, available on
`https://math.temple.edu/~seibold/research/particleclaw/` to solve the LWR problem given above. Email your program `yourfamilyname_problem5a.m` (that can be run together with the *particleclaw* solver file) that produces an animation of the solution on $x \in [-1, 8]$ for $t \in [0, 20]$.

b) Find the true solution of the problem at $t = 20$.

c) Start with the resolution `parameters.d = [0 1e-1 2e-1]` . Then successively refine the resolution by powers of 2. For each computation, evaluate the L^1 error. Visualize the error convergence rate, by plotting (in log-log scale) the L^1 error vs. the third argument of `parameters.d = [0 1e-1 2e-1]` . Email your program `yourfamilyname_problem5c.m` .

Problem 6

For the same equation as in Problem 5, we now wish to compute the solution using the finite volume software *Clawpack*. The software, and many forms of documentation, are available on `http://www.clawpack.org` .

You need a Unix/Linux-type environment to run *Clawpack*. Under Windows, you can use *Cygwin*. There is also a virtual environment for running *Clawpack*. Please read the installation instructions on
`http://www.clawpack.org/installing.html` carefully, and make your own choices about how you prefer to install/run the software.

Once installed, you probably want to run one or more existing examples
(see `http://www.clawpack.org/first_run.html`) to get a lay of the land.

After that, you may want to find an existing example that is very close to the LWR equation, and adapt it according to your needs. Then:

a) For a resolution of $\Delta x = 0.1$, produce a plot of the *Clawpack* solution at time $t = 20$.

c) *[Optional, 10 bonus points]* Conduct a numerical error convergence analysis of the *Clawpack* method (using a high-order solver), measuring errors in the L^1 norm.