Temple 9200 **Computational Methods for Flow Problems** **Spring 2015**

Problem Set 4

(Out Wed 02/25/2015, Due Wed 03/11/2015)

Problem 4

Consider the 1d advection-reaction equation

$$\phi_t + \phi_x = r(\phi) + s(x, \phi)$$

on $x \in [0, 2\pi[$ with periodic boundary conditions, and zero initial conditions $\phi(x, 0) = 0$. The solution $\phi(x, t)$ represents a chemical concentration ($0 \leq \phi \leq 1$), which is advected with constant velocity, and modified by a bistable reaction term $r(\phi) = \phi(1-\phi)(\phi - \frac{1}{2})$ and a localized source term $s(x, \phi) = a \exp\left(-10(x-\pi)^2\right)(1-\phi)$, where $a > 0$ is a parameter.

(1) Write a Matlab program that approximates the true solution with sufficient accuracy, and run the simulation on the two cases $a = 0.5$ and $a = 1$. Email your code under the file name `yourfamilyname_problem4a.m` . Moreover, in your paper submission, plot both solutions at times $t \in \{2, 8, 40\}$. Explain your observations.

(2) There is a critical threshold value a_c, such that for $a < a_c$, the solution behaves like the case $a = 0.5$, and for $a > a_c$, the solution behaves like the case $a = 1$. Find a_c numerically, up to at least 0.1% accuracy. Remember that your scheme's global truncation error must be sufficiently small.
Competition: Whoever gets the closest to the true value of a_c (or finds this value analytically) receives 10 bonus points.