Temple 9200 **Computational Methods for Flow Problems** **Spring 2015**

Problem Set 3

(Out Wed 02/11/2015, Due Wed 02/25/2015)

Problem 3

Consider the same 1d advection-reaction equation as in problem set 2:

$$\phi_t + u\phi_x = g(\phi)$$

on the domain $x \in [-1, 1]$, where the flow velocity field is $u(x) = \sin(2\pi x)$, the reaction term is $g(\phi) = -6(\phi - 1)\phi(\phi + 1)$, and the initial state is $\phi(x, 0) = \sin(\pi x)$. We are interested in the solution at $t = 3$.

(a) Derive and program a first-order semi-Lagrangian scheme (using backwards tracking) to solve this problem. Use simple Euler steps for the characteristic ODEs, and simple piecewise linear interpolation (you can use Matlab's `interp1` function with the option `'linear'`). Explain in your paper submission how the scheme works, and email your code under the file name `yourfamilyname_problem3a.m` . Choose the grid spacing Δx and time step Δt so that the numerical approximation is correct in the "eye norm", i.e., it deviates from the true solution by about 1%.

(b) Modify your code to yield a third-order semi-Lagrangian scheme. To that end, replace the Euler steps by RK3 steps, and change the piecewise linear interpolation to a piecewise cubic (e.g., by using `interp1` with the option `'cubic'` [1]). Choose the grid spacing and time step so that the numerical approximation is correct in the "eye norm", and submit your code under the file name `yourfamilyname_problem3b.m` .

(c) Choose the grid spacing proportional to the time step (here: $\Delta x = \Delta t$) and conduct a numerical convergence study with your two semi-Lagrangian schemes. To that end, choose a sequence of resolutions, and for each choice of Δt determine the error of the numerical approximation $e = E(\Delta t)$. Then plot the function $E(\Delta t)$ in log–log scale and read off the convergence rate as its slope (for the small values of Δt). Submit your code under the file name `yourfamilyname_problem3c.m` , and submit a figure of the convergence plot in your paper submission.

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[1] As an optional side problem, try to understand what the difference between the options `'cubic'` and `'spline'` is; and argue which choice is preferable for the task at hand.