Temple 3044 Numerical Analysis II Spring 2012

Problem Set 10

(Out Thu 04/12/2012, Due Thu 04/19/2012)

Instructions

- Problems marked with **(T)** are theory problems. Their solutions are to be submitted on paper.
- Problems marked with **(P)** are practical problems, and require the use of the computer. Their solutions are to be submitted on paper, and usually require two parts: (a) a description of the underlying theory; and (b) code segments, printouts of program outputs, plots, and whatever it required to convince the grader that you have understood the theory and addressed all practical challenges appropriately.

Generally, naked numbers are not acceptable. Solutions must include a short write-up describing the problem, your solution technique, and procedural details. To include a computer printout use the cut and paste method for placement of materials in your work. All things must be clearly labeled.

Problem K

(P) Consider the advection-reaction equation

$$u_t + u_x = r(u) + s(x, u)$$

on $x \in [0, 2\pi[$ with periodic boundary conditions, and zero initial conditions $u(x, 0) = 0$. The solution $u(x, t)$ represents a chemical concentration ($0 \leq u \leq 1$), which is advected with constant velocity, and modified by a bistable reaction term $r(u) = u(1-u)(u-\frac{1}{2})$ and a localized source term $s(x, u) = a \exp\left(-10(x-\pi)^2\right)(1-u)$, where $a > 0$ is a parameter.

(1) Write a program that approximates the true solution with sufficient accuracy, and run the simulation on the two cases $a = 0.5$ and $a = 1$. Plot both solutions at times $t \in \{2, 8, 40\}$. Explain your observations.

(2) There is a critical threshold value a_c, such that for $a < a_c$, the solution behaves like the case $a = 0.5$, and for $a > a_c$, the solution behaves like the case $a = 1$. Find a_c numerically, up to at least 0.1% accuracy. Remember that your scheme's global truncation error must be sufficiently small.