Temple 3044 Numerical Analysis II Spring 2012

Problem Set 8

(Out Thu 03/29/2012, Due Thu 04/05/2012)

Instructions

- Problems marked with **(T)** are theory problems. Their solutions are to be submitted on paper.
- Problems marked with **(P)** are practical problems, and require the use of the computer. Their solutions are to be submitted on paper, and usually require two parts: (a) a description of the underlying theory; and (b) code segments, printouts of program outputs, plots, and whatever it required to convince the grader that you have understood the theory and addressed all practical challenges appropriately.

Generally, naked numbers are not acceptable. Solutions must include a short write-up describing the problem, your solution technique, and procedural details. To include a computer printout use the cut and paste method for placement of materials in your work. All things must be clearly labeled.

Problem J

(P) Download the program `temple3044_heateqn.m` from the course web site.

(a) Run it, and explain which problem it solves, and how.

(b) Extend the program to allow also for using backward Euler and Crank-Nicolson time stepping. Use the plain Matlab backslash to solve linear systems.

(c) For all three versions of time stepping, plot the numerical solution at the final time for the parameter choice $h = \frac{1}{30}$ and $\Delta t = 5.6 \cdot 10^{-4}$. Watch the animation, and explain your observations.

(d) Compute a reference solution, using lots of grid points.

(e) For the three versions of time stepping, perform a numerical error and performance analysis, as follows. For the following approaches

(A) forward Euler with $\Delta t = \frac{1}{2}h^2$

(B) backward Euler with $\Delta t = \frac{1}{2}h^2$

(C) Crank-Nicolson with $\Delta t = \frac{1}{2}h^2$

(D) forward Euler with $\Delta t = \frac{1}{2}h$

(E) backward Euler with $\Delta t = \frac{1}{2}h$

(F) Crank-Nicolson with $\Delta t = \frac{1}{2}h$

compute the L^∞ error with respect to the reference solution, as well as the computational time[1] for a sequence of mesh resolutions $h \in \{\frac{1}{10}, \frac{1}{20}, \frac{1}{40}, \frac{1}{80}, \frac{1}{160}\}$. Plot the error and compute time curves as functions of h in double-log scale.

[1] Measured in seconds using `tic` and `toc`; remove any plotting routines for this investigation.