

Problem Set 2

(Out Thu 01/26/2012, Due Thu 02/02/2012)

Instructions

- Problems marked with **(T)** are theory problems. Their solutions are to be submitted on paper.
- Problems marked with **(P)** are practical problems, and require the use of the computer. Their solutions are to be submitted on paper, and usually require two parts: (a) a description of the underlying theory; and (b) code segments, printouts of program outputs, plots, and whatever it required to convince the grader that you have understood the theory and addressed all practical challenges appropriately.

Generally, naked numbers are not acceptable. Solutions must include a short write-up describing the problem, your solution technique, and procedural details. To include a computer printout use the cut and paste method for placement of materials in your work. All things must be clearly labeled.

Problem A

(P) Write a Matlab program that uses adaptive quadrature with a three-point Gaussian quadrature rule (see http://en.wikipedia.org/wiki/Gaussian_quadrature for the abscissae and weights) to approximate

$$\int_0^2 e^x \sin(x^2 \cos(e^x))$$

up to 10 decimal places (see also Bradie Example 6.19).

Problem B

(T)&(P) Consider a Lotka-Volterra predator-prey model for a population of carps and pikes, whose numbers are given by $x(t)$ and $y(t)$, respectively. The growth/decay rates are given by

$$\begin{aligned} \frac{dx}{dt} &= x - 4xy \\ \frac{dy}{dt} &= -y + 5xy \end{aligned} \tag{1}$$

(a) Show that the function $H(x, y) = xy \exp(-5x - 4y)$ is a *constant of motion*, i.e. if $(x(t), y(t))$ is a solution of (1), then $H(x(t), y(t))$ is constant in time.

(b) Using Matlab's `mesh` command, plot the function H on the domain $(x, y) \in [0, 1]^2$.

(c) Using Matlab's `quiver` command, plot the velocity field given by the right hand side vector of (1), scaled to length 1 everywhere. On top of this plot, overlay isocontours of the function H , using Matlab's `contour` command.

(d) Starting with $x(0) = 0.2$ and $y(0) = 0.8$, approximate (1) using Euler's method (see Bradie 7.2) for $t \in [0, 8]$. Use steps of size $\Delta t = 0.01$. Plot all 801 points obtained from this numerical solution into the figure created in (c).