Temple 3043 Numerical Analysis I Fall 2011

Problem Set 7

(Out Mon 10/17/2011, Due Tue 10/25/2011)

Instructions

Any problem given by a number (and page reference) is taken from the book
Brian Bradie, *A Friendly Introduction to Numerical Analysis*, Pearson Prentice Hall, 2006.

- Problems marked with **(T)** are theory problems. Their solutions are to be submitted on paper.
- Problems marked with **(P)** are practical problems, and require the use of the computer. Their solutions are to be submitted on paper, and usually require two parts: (a) a description of the underlying theory; and (b) code segments, printouts of program outputs, plots, and whatever it required to convince the grader that you have understood the theory and addressed all practical challenges appropriately.

Problem B

The fundamental error estimates on p. 182 and p. 184 were derived for a vector norm $\|\cdot\|_v$ and the associated matrix norm, defined by $\|A\| = \max_{x\neq 0} \frac{\|Ax\|_v}{\|x\|_v}$. For the Euclidian vector norm $\|\cdot\|_2$, the associated matrix norm is complicated and costly to evaluate. We would like to know whether one can instead use the simpler to compute Frobenious matrix norm $\|A\|_F = \sqrt{\sum_{i,j=1,\ldots,n} |a_{ij}|^2}$ in the estimates. Show the following:

(a) An associated matrix norm must satisfy $\|I_n\| = 1$, where I_n is the $n \times n$ identity matrix.

(b) The Frobenious norm is not an associated norm.

(c) Nevertheless, the Frobenious norm and the Euclidian vector norm satisfy $\|Ax\|_2 \leq \|A\|_F \|x\|_2 \ \forall A, x$, i.e. the error estimates in fact hold for these norms.

(d) What is the disadvantage of using the Frobenious norm instead of the 2-matrix-norm in the error estimates?

Section 3.4 (pages 187–190)

(T) 3. **(T)** 11. **(P)** 13.

Section 3.5 (pages 201–204)

(T) 3. **(T)** 8.
(P) 15. (extend your Matlab routines for elimination to compute the LU factorization with pivoting.