

Problem Set 2

(Out Tue 02/09/2010, Due Tue 02/23/2010)

Problem 4

The van der Pol oscillator

$$u'' - \mu(1 - u^2)u' + u = 0 \quad (1)$$

is a linear oscillator (case $\mu = 0$), augmented by a nonlinear damping¹ term.

- (1) Someone claims that for any $\mu > 0$, (1) has a stable limit cycle solution. Please verify this numerically, and plot the limit cycles (in the u - u' -plane) for $\mu \in \{\frac{1}{2}, 1, 2, 4, 8\}$.
- (2) For any $\mu \geq 0$, the periodic solution has a given period T . Perform a sequence of numerical runs for various values of μ , and plot the function $T(\mu)$, for $\mu \in [0, 50]$.
- (3) Contest: Who gets the largest value of μ , for which you find $T(\mu)$ numerically?

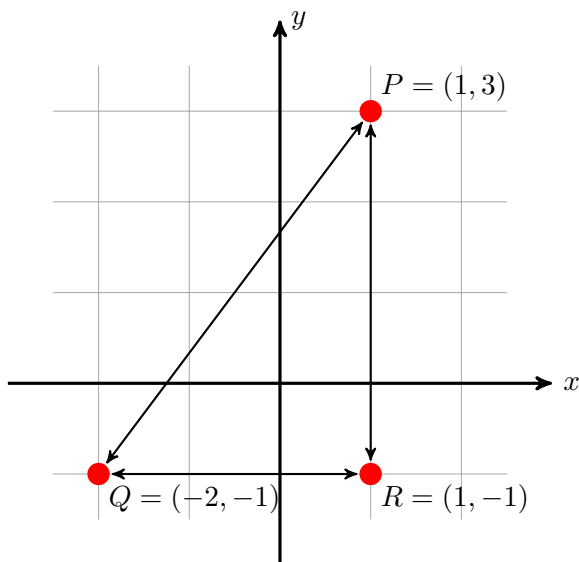
Problem 5

A supernatural creator² places three planets P , Q , R with masses 3, 4, 5 at the positions shown in the figure to the right. At time $t = 0$, the planets are at rest. The laws of motion are given by pairwise gravitational attraction $F = \frac{m_1 m_2}{d^2}$.

- (1) Set up a first order system of ODE that describes the dynamics.
- (2) Compute the solution of the arising system of ODE up to time $T = 75$, and plot the trajectories of the three planets in the xy -plane.
- (3) Explain your observations.

Some remarks:

1. This problem is computationally challenging. You may need a high order ODE solver with adaptive time stepping to get close to the true solution.
2. You can check whether you are close to the true solution by diving the time step in half and comparing the result.



¹Here, the damping can reduce as well as increase the energy of the system.

²Insert your favorite example