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Effect of the Choice of Jam Density in Data-Fitted First- and Second-Order Traffic Models

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Abstract

For a class of data-fitted macroscopic traffic models, the influence of the choice of the jam density on the model accuracy is investigated. This work builds on an established framework of data-fitted first-order Lighthill-Whitham-Richards (LWR) models and their second-order Aw-Rascle-Zhang (ARZ) generalizations [4]. These models are systematically fitted to historic fundamental diagram data, and then their predictive accuracy is quantified via a version of the three-detector problem test, considering vehicle trajectory data and single-loop sensor data. The key outcome of this study is that with commonly suggested jam densities of 120 vehicles/km/lane and above, information travels backwards too slowly. It is then demonstrated that the reduction of the jam density to 90–100 vehicles/km/lane addresses this problem and results in a significant improvement of the predictive accuracy of the considered models.

First-Order Models vs. Second-Order Models

Macroscopic traffic modeling: describe the collective vehicle dynamics in terms of aggregate traffic density $\rho(x,t)$, traffic flow rate $Q(x,t)$, and average velocity $u(x,t) = Q(x,t)/\rho(x,t)$. This approach results in (systems of) **hyperbolic conservation laws**.

- First-order LWR model [1]: a scalar mass conservation equation

$$\rho_t + (Q(\rho))_x = 0\,, \quad \text{where} \quad Q(\rho) = \rho U(\rho).$$

  The flow-density function $Q(\rho)$ is a fundamental diagram (FD).
- Second-order model: a system of conservation laws, e.g., the ARZ model [2, 3]

$$\begin{cases} \rho_t + (\rho u)_x = 0 \\ w_t + u w_x \;\; = 0\,, \end{cases}$$

  where w represents a property of drivers that is advected with the vehicles. In the ARZ model, $w = u + (U(0) - U(\rho))$ is the empty road velocity of drivers.

The ARZ model is a generalization of the LWR model in the sense that ARZ allows different drivers to have different properties.

Data-Fitted First- and Second-Order Models

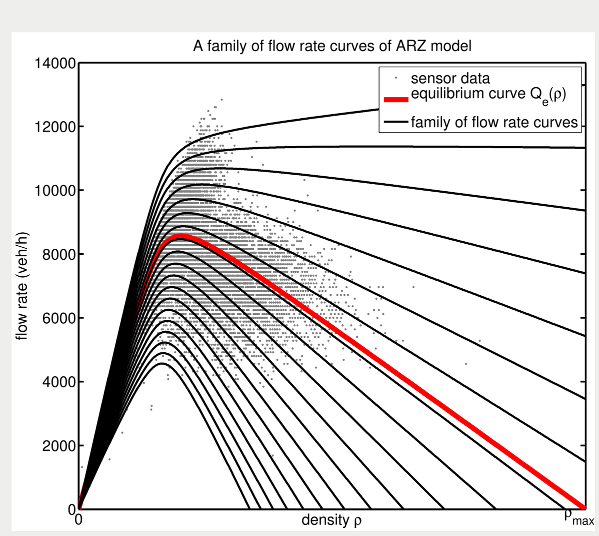
The LWR model employs a single flow rate curve $Q(\rho)$ [red curve]. This induces a family of flow rate curves [black curves] in the ARZ model

$$Q_w(\rho) = Q(\rho) + \rho\,(w - U(0))\,.$$

Data-Fitting Methodology [4]:

- Use historic FD data (ρ_j, Q_j) to construct data-fitted macroscopic models.
- Prescribe a flow rate function with free parameters, e.g., a 3-parameter model $Q_{\alpha,\lambda,p}(\rho)$.
- Let the jam density $\rho_{\max}$ be a fixed model parameter.
- Identify free parameters by a LSQ fit with data

$$\min_{\alpha,\lambda,p} \left\{ \sum_{j=1}^{n} \left(Q_{\alpha,\lambda,p}(\rho_j) - Q_j\right)^2 \right\}.$$

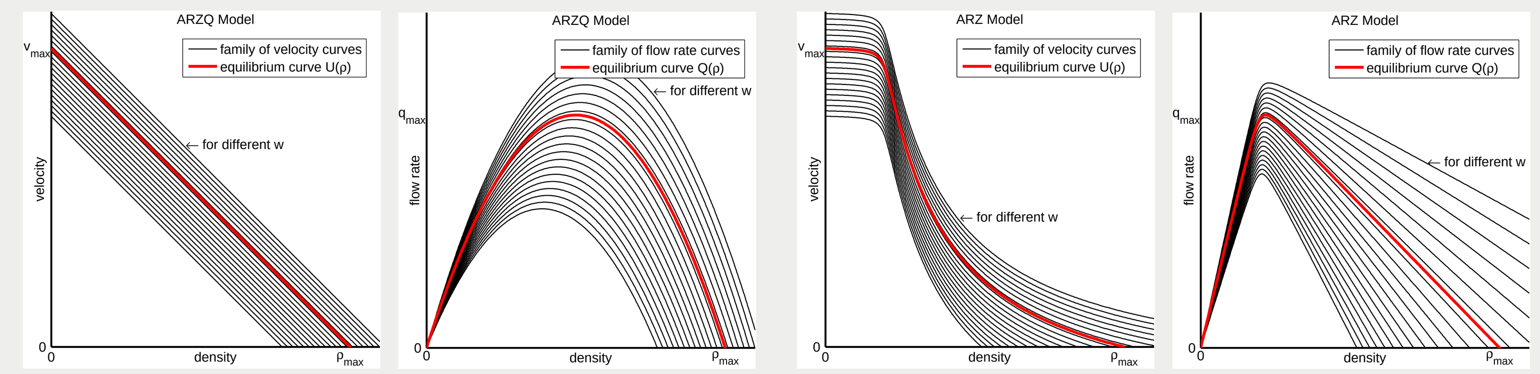


Wave Propagation Speeds in Traffic Models

- **LWR:** characteristic speed: $\lambda = Q'(\rho)$; shock wave speed: $s = [Q(\rho)]/[\rho]$.
- **ARZ:** slower characteristic field: $\lambda_1 = Q_w'(\rho)$ and $s = [Q_w(\rho)]/[\rho]$; faster characteristic field: $\lambda_2 = u$ and no shocks (only contact discontinuities).
- **Shown below:** $\rho_{\max}$ has substantial effect on the travel speed of information.

Velocity and FD Curves for Various Traffic Models

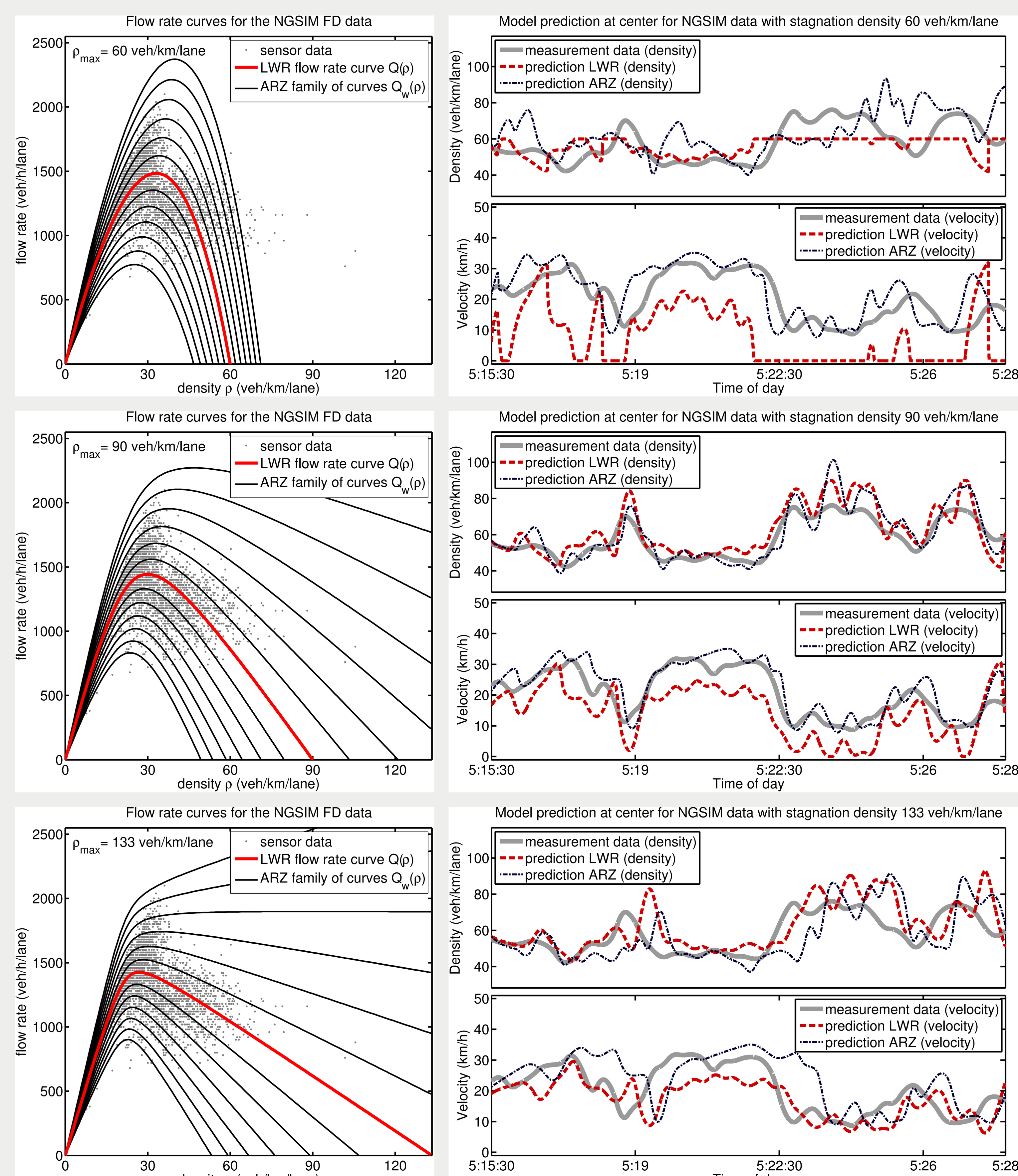
Greenshields flux ⇝ LWRQ & ARZQ **New 3-parameter flux ⇝ LWR & ARZ**



Velocity and flow rate curves for data-fitted models. Red curves: first-order models LWRQ and LWR. Black curves: second-order generalizations ARZQ and ARZ.

Effective of Choice of Jam Density: Wave Speeds

Model validation using NGSIM trajectory data [6]. Left column of figures: data-fitted flow rate curves for three different jam densities $\rho_{\max} \in \{60, 90, 133.33\}$ veh/km/lane. Right column of figures: temporal evolution of the true densities and velocities (gray), and the corresponding quantities predicted by the models LWR (red) and ARZ (black).



Observations:

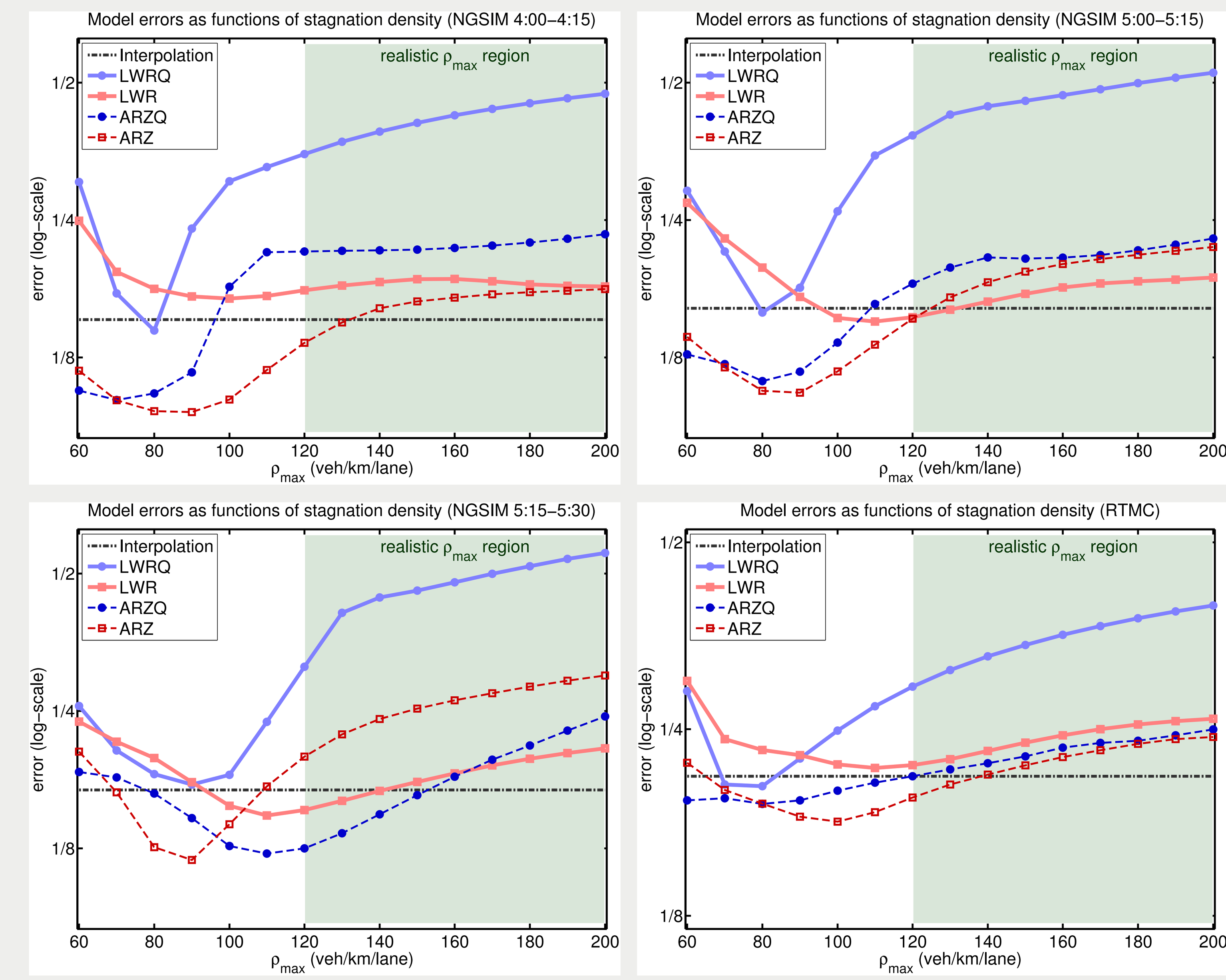
- ARZ shows better agreement with real traffic data than LWR.
- Models with $\rho_{\max} = 90$ veh/km/lane capture the sudden transition from free flow to congestion most accurately.
- Validation results obtained with loop sensor data [5] confirm these observations.

Model Accuracy as a Function of the Jam Density

Error Definition:

$$e(x,t) = \frac{\left|\rho^{\text{data}}(x,t) - \rho^{\text{model}}(x,t)\right|}{\Delta\rho} + \frac{\left|u^{\text{data}}(x,t) - u^{\text{model}}(x,t)\right|}{\Delta u}\,.$$

Here, $\Delta\rho$ and Δu are the maximum variation in density and velocity that the historic FD data exhibits, both modulo outliers. The figures show the space-time-averaged error $\bar{e} = \frac{1}{(x_2-x_1)}\frac{1}{(t_2-t_1)}\int_{t_1}^{t_2}\int_{x_1}^{x_2} e(x,t)\,dx\,dt$ as a function of the jam density $\rho_{\max}$.



Observations:

- Each of the four traffic models possesses an optimal jam density $\rho_{\max}^{\text{opt}}$ for which the error is minimized, i.e., the model reproduces real traffic behavior best.
- In all cases, this $\rho_{\max}^{\text{opt}}$ lies clearly below the range of supposedly realistic values ($\rho_{\max} \geq 120$ veh/km/lane), for which information would propagate too slowly.

Conclusions

- ARZ more accurate than LWR; 3-param. flux better than Greenshields flux [4].
- The choice of the jam density $\rho_{\max}$ has a significant effect on the wave propagation speed in congestion.
- The dependence of the overall errors on $\rho_{\max}$ indicates that choosing $\rho_{\max} \in [90, 100]$ veh/km/lane yields more accurate macroscopic models than choices of $\rho_{\max} \in [120, 200]$ veh/km/lane, as suggested in the literature.
- However, clearly $\rho_{\max} > 120$ veh/km/lane when cars are bumper-to-bumper. This may hint at the need for non-convex fundamental diagrams that are actually convex up for highly congested traffic!

References

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