Dynamics of Balls and Liquid in a Ball Mill

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Abstract

We consider a cylindric ball mill, filled with small hardmetal balls and a suspension. By rotating the cylinder around its main-axis the suspension shall be mixed by the balls, which are lifted up on one side and then roll or fall back onto their own surface. The lifting is increased by nine steel bars along the cylinder wall. The problem is that the balls must not have too high velocities when they hit their surface, because they can be damaged or even broken. Therefore it is necessary to describe the movement and the surface structure of the balls under the influence of the suspension. We model the dynamics inside the ball mill in dependence of the filling volumes of suspension and balls by first considering only balls without suspension and then modeling the suspensions' influence. From this we get estimates for the energy of balls hitting their surface, which is an important value for the company.

Contents

Chapter 1

Introduction

1.1 Mixing process in a rotating ball mill - an overview

United Hardmetal GmbH is a subsidiary of the international Cerametal Group, Luxembourg. The company is an important supplier for cemented carbide products[1]. As a stage in the technological process [2], mixing of the metal carbide (tungsten-carbide) with the binder material (cobalt) takes place in a so-called "ballmill". In this case it is a rotating cylinder at 150 *cm* length and 80 *cm* diameter shown in Figure 1.1, having the wall covered with liners with 1 *cm* height and 1.5 *cm* width and the length of the cylinder.
The ballmill is filled with:

- *hardmetal balls* of 12 *mm* diameter, having a density of 14.5 g/cm^3, and a total mass of approximately 3000 *kg* ($\approx$ 230000 *balls*)
- *grinding suspension* (tungsten-carbide (WC), cobalt, paraffin, alcohol) of density 2 $g\ cm^{-3}$, of volume between 300 *l* and 400 *l*.

Rotating the cylinder with 22 *rpm*, the balls are lifted up on left side with the influence of the steel bars, which prevent the balls from sliding down the cylinder wall, and then fall down into their own surface, helping to mix the material.

[1]Cemented carbides are produced by mixing various metal carbides such as tungsten carbide, tantalum carbide, vanadium carbide, niobium carbide, chromium carbide, molybdenum carbide and/or tungsten/titanium carbide with a binder material which is usually cobalt but can be nickel or a combination of nickel and cobalt. The binder is added as a percentage by weight varying from 3% to 30%. The amount of binder used is a very important factor in determining the properties of each grade. As a rule of thumbs the lower the cobalt content the harder the material will become. However variation in grain size and additives can upset this rule.

[2]This mixture is generally held together by some type of organic binder and formed into a desired shape. After the forming operation, the material is sintered in a furnace. The sintering process melts the binder material around the carbide particles. In the process of sintering, the material shrinks volumetrically about 43%. After sintering, the material is generally ground to the final dimensions before being placed into service.

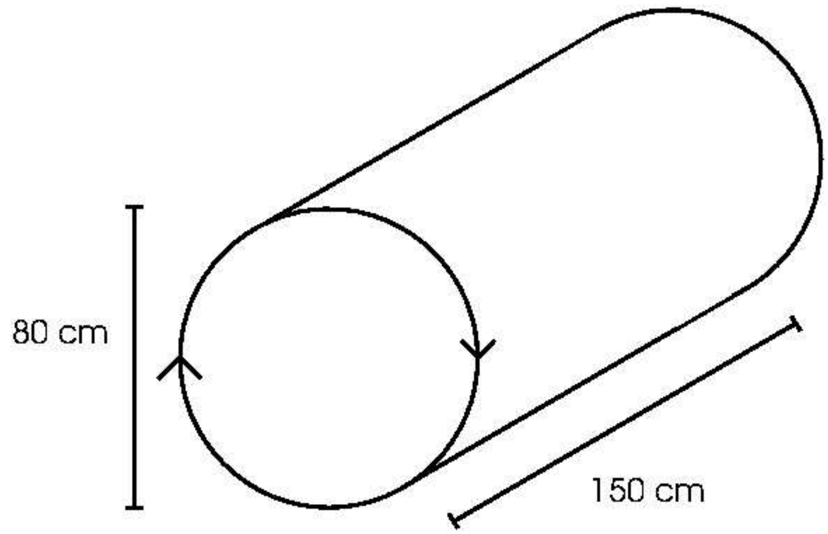


Figure 1.1: Outer dimensions of the cylinder

The aim of this project is to find a mathematical model for the movement of ball-suspension-mixture during the rotation and to give answers to the following questions that are of great interest for the company:

1. What is the movement of the balls during the rotation depending on the degree of filling of the mill?
2. Which surface does the ball-suspension mixture have during the rotation?
3. To which height do the balls move along the wall and along which curves do they fall back into the mixture?
4. What is the energy of the balls when they hit the surface of the filling?

Since the company is interested in the durability of the balls and the cylinder wall, we concentrated in developing a model for the movement of the balls in order to calculate the velocities and energies of the balls when they hit their own surface or the cylinder wall.

From literature we know three different types of movement in the ball mill during rotation:

a) *Slow Rotation ("Cascading", Figure 1.2):*

   (a) the balls are not lifted up very high
   (b) they roll from each other
   (c) they do not fly around

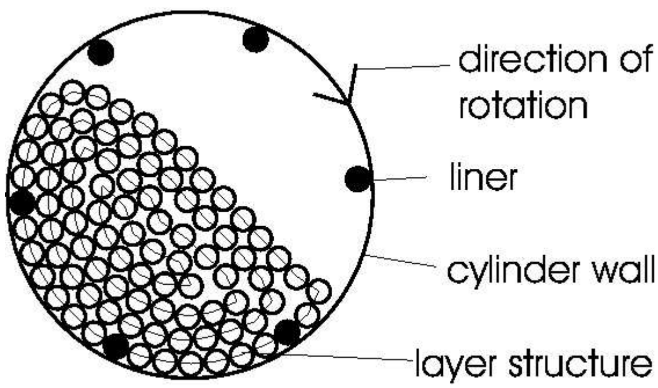


Figure 1.2: Cascading

b) *Fast Rotation ("Cataracting", Figure 1.3):*

   (a) the balls are lifted up very high
   (b) they lose contact with the cylinder wall
   (c) they fall back in parabola-like trajectories

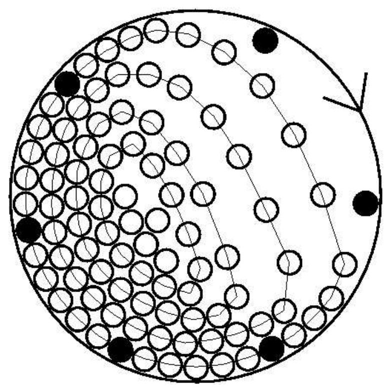


Figure 1.3: Cataracting

c) *Very Fast Rotation ("Centrifugation", Figure 1.4):*

   (a) the balls are lifted up

   (b) they do not lose contact with the cylinder wall (the centrifugal force is bigger than the gravitational force)

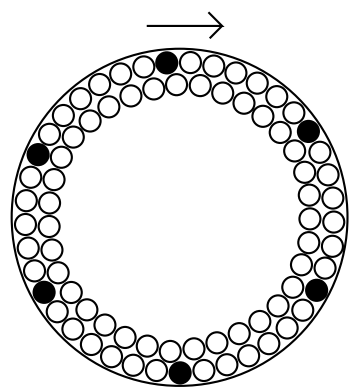


Figure 1.4: Centrifugation

1.2 An impression about the important effects

As a first glance to the problem we compute the filling heights of the mill with only balls (dry milling), and balls with suspension for two different cases: 300 l and 400 l of suspension and a packing of the balls of 75% (25% free space between the balls). These filling heights are plotted in Figure 1.5 where h_1 is the filling height with only balls and h_2 and h_3 are the filling heights with balls and 300 l and 400 l respectively.
For the general case of movement of balls in suspension, we estimate the forces acting on a ball:

Gravitational force: $F_G \approx 13\ cN$ (determined exactly by the mass of a ball and the gravitational constant)

Centrifugal force: $F_C \approx 2.3\ cN$ (depends on the distance from the middle of the cylinder, the value taken here is for the outermost layer and by this it is an upper bound)

Buoyancy force: $F_A \approx 1.8\ cN$ (determined exactly by the volume of suspension replaced by a ball)

Stokes' force: $F_S \approx 1.9\ cN$ (depends on the velocity of a ball, the value taken here is for the balls in the outermost layer and by this it is again an upper bound)

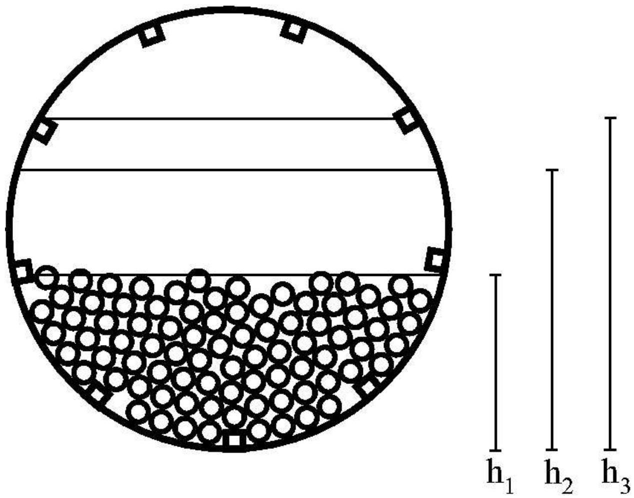


Figure 1.5: Filling heights ($h_1 \approx 32\ cm$, $h_2 \approx 51\ cm$, $h_3 \approx 60\ cm$)

Considering all these facts, we can conclude that none of these forces can be neglected, so, in all our further models we take all of them into account.

1.3 Considering the problem

In order to answer the questions posed by the company, we first modeled the movement of balls in the ball mill without influence of suspension ("dry milling"). In our second model we calculated the movement of balls in the resting suspension. In the third model we considered the movement of suspension alone in order to calculate the velocity field of the suspension. We then computed the movement of the balls under the influence of the moving suspension.
Even if the rotation speed of the company's ball mill is considered to be fixed, we consider in developing our model the rotation speed as a parameter, as well as volume of suspension and the number of balls.

Chapter 2

Dry Milling

2.1 Basic ideas and assumptions

In this section we introduce the first model. This is called dry milling because we consider only the movement of hardmetal balls in the cylinder and neglect the influence of the suspension. By this we only have to consider two forces, namely the gravitational force and the centrifugal force. By that we can easier describe the movement of the balls. For simplification we introduce further assumptions:

1. We use a two dimensional model with balls as equal-sized discs and the filling height calculated from the three dimensional model. This is reasonable because we have nearly the same filling height everywhere in the cylinder and the movement of the balls is mainly in the direction of the rotation of the mill and not in the direction of the main axis.

2. We consider a perfect layer structure. This means that the balls are lifted up in layers with distance of twice the radius of a ball and with the angular velocity of the mill. We can do so because the first layer is lifted up by the steel bars with the angular velocity of the cylinder and the other layers are pressed to the outer layers and so they cannot slide down and are taken with them. This structure is shown in Figure 2.1 for five layers.

3. For the calculation of the surface structure we consider a "steady state" which means that the balls are moving but their surface structure does not change.

4. We neglect the rotation of the balls itself.

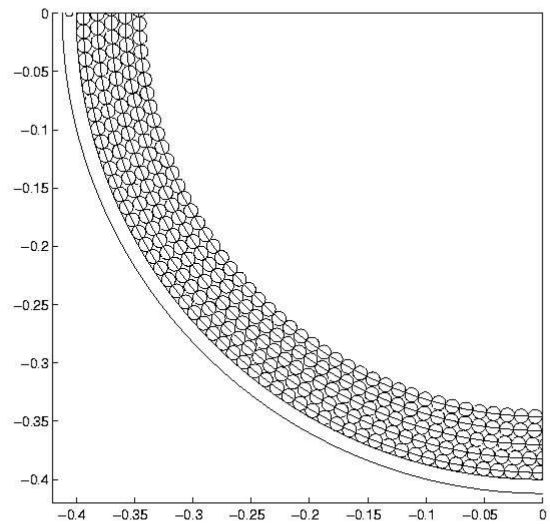


Figure 2.1: Perfect layer structure

2.2 Movement of the balls

In this section we calculate the movement of the balls which are lifted up by the steel bars. Because of these steel bars we can assume that there is no slip between the hardmetal balls and the cylinder wall. We consider the forces acting on one ball in the first layer (see Figure 2.2):

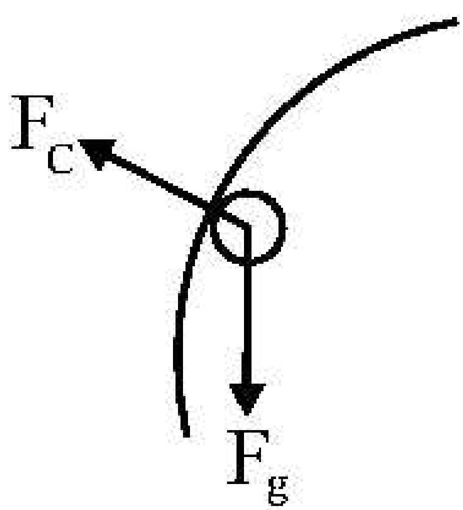


Figure 2.2: Forces acting on a ball

$F_C = m(R - r)\omega^2$ is the centrifugal force and
$F_g = mg$ is the gravitational force

with

m = mass of one hardmetal ball = 13.12 g,
R = radius of the cylinder = 0.4 m,
r = radius of a ball = 0.006 m,
ω = angular velocity = $2\pi \cdot \frac{22}{60}\ s^{-1}$,
g = gravitational constant = 9.81 ms^{-2}.

In the following we calculate the movement of balls in three different regions (see Figure 2.3)

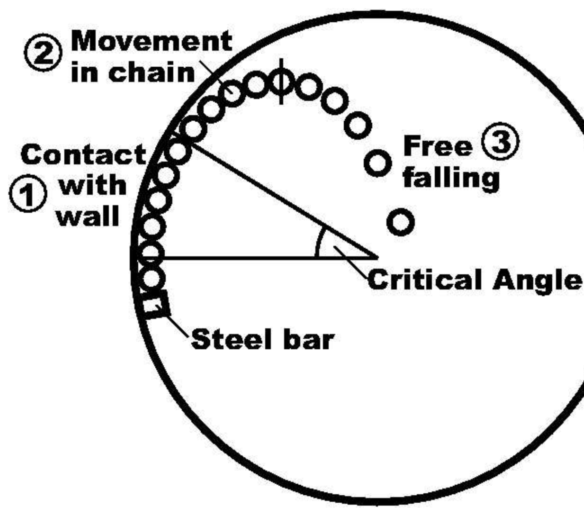


Figure 2.3: The three different types of movement

and decompose the centrifugal force and the gravitational force into a radial and a tangential direction
with

$$F_r = m(R-r)\omega^2 - mg\sin(\alpha) \qquad \text{(radial force)},$$
$$F_t = mg\cos(\alpha) \qquad \text{(tangential force)}.$$

In region ① in Figure 2.3 the balls are lifted up by the steel bars and must move with the same velocity as the cylinder. The radial component points out of the cylinder because of the bigger influence of the centrifugal force and so the balls are pressed to the cylinder wall and cannot lose contact with it. So the balls follow a circular path with distance $(R-r)$ from the origin. Having a glance on the radial force F_r gives that this force reduces with α increasing. By this we can calculate the angle α_c where the radial force equals zero:

$$\alpha_c = \arcsin \frac{(R-r)\omega^2}{g}.$$

This is called the critical angle. At this angle the radial force is equal to zero and with a further increase of this angle the radial force changes the direction and points to the

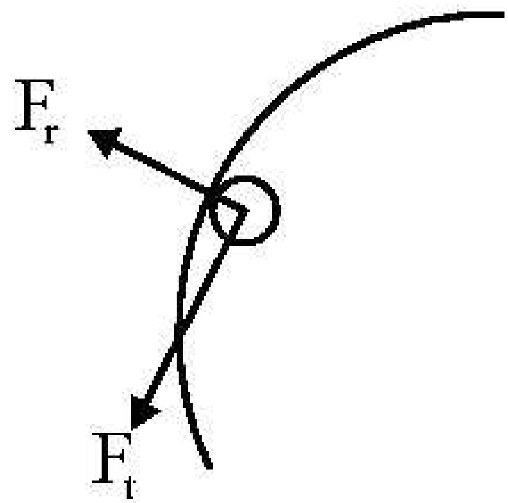


Figure 2.4: Radial and tangential force

origin. Up to this point the ball moves with the angular velocity

$$\omega = 2 \cdot \pi \cdot \frac{22}{60} \quad s^{-1}.$$

By this we get the velocity vector of each ball in the first layer

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = (R - r)\omega \begin{pmatrix} \sin\alpha \\ \cos\alpha \end{pmatrix}. \tag{2.1}$$

Using the assumption of the perfect layer structure we get for the k-th layer the critical angle

$$\alpha_{ck} = \arcsin \frac{(R - (2k - 1)r)\omega^2}{g}$$

and the velocity vector

$$\vec{v_k} = \begin{pmatrix} v_{1k} \\ v_{2k} \end{pmatrix} = (R - (2k - 1)r)\omega \begin{pmatrix} \sin\alpha \\ \cos\alpha \end{pmatrix}.$$

Considering again the first layer:
If the angle α surpasses the critical angle the ball enters region ② in Figure 2.3. Here the ball loses contact with the cylinder wall because the direction of the radial force changes. In this region we have a movement in a chain because the ball wants to fall down because of the gravitational force but is lifted up by the balls behind which are lifted up by the rotation of the mill.
The gravitational acceleration can be decomposed into two perpendicular vectors $\vec{a}$ and $\vec{b}$ where the vector $\vec{a}$ itself is perpendicular to $\vec{v}$. The acceleration $\vec{b}$ is taken by the balls behind (and so by the cylinder wall or the steel bar at the end of the chain) an has no

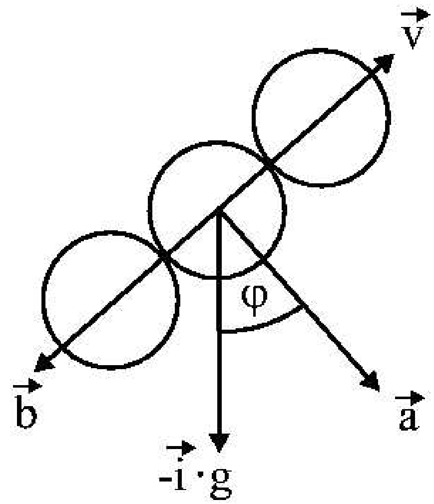


Figure 2.5: Forces in the movement in a chain

influence in this region. The movement of the ball can be calculated by the velocity $\vec{v}$ and the acceleration $\vec{a}$ which is depending on the angle φ. So we need $\vec{a} = \vec{a}(\varphi)$.
Taking $\vec{i}$ as the unit vector in x-direction and $\vec{j}$ as the unit vector in y-direction and $\vec{l}$ as the unit vector in $\vec{a}$-direction we get

$$\langle \vec{v}, \vec{i} \rangle = \|\vec{v}\| \cdot \|\vec{i}\| \cdot \cos\varphi = \|\vec{v}\| \cos\varphi$$

$$\Rightarrow \qquad \cos\varphi = \frac{\langle \vec{v}, \vec{i} \rangle}{\|\vec{v}\|}.$$

Using $\langle \vec{v}, \vec{i} \rangle = v_1$ and $\|\vec{v}\| = (v_1^2 + v_2^2)^{\frac{1}{2}}$ we get

$$\cos\varphi = \frac{v_1}{(v_1^2 + v_2^2)^{\frac{1}{2}}}.$$

Taking $\vec{j}$ we get analogously

$$\sin\varphi = \frac{v_2}{(v_1^2 + v_2^2)^{\frac{1}{2}}}.$$

For the vector $\vec{l}$ there holds

$$\vec{l} = -\vec{j}\cos\varphi + \vec{i}\sin\varphi = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix}$$

Using the equation for $\cos\varphi$ and $\sin\varphi$ and the fact that $\vec{a} = g\cos\varphi \cdot \vec{l}$ we finally get

$$\frac{d}{dt}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = g \begin{pmatrix} \frac{v_1 v_2}{v_1^2+v_2^2} \\ \frac{-v_1^2}{v_1^2+v_2^2} \end{pmatrix}. \qquad (2.2)$$

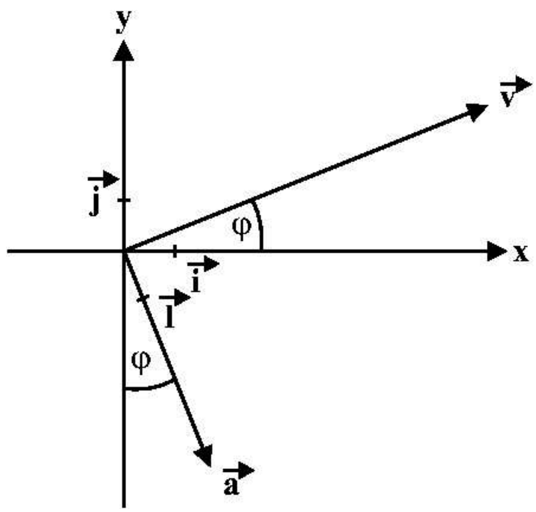


Figure 2.6: The vectors in a coordinate system

By solving this nonlinear explicit 2-dimensional system of ODE's we obtain the trajectory for the balls in region ②in Figure 2.3. This movement is not depending on the number of the layer and ends when the vector $\vec{b}$ in Figure 2.5 is equal to zero. This is the case when v_2 is equal to zero. When this point is reached we enter region ③ in 2.3. Here the ball is not connected to the balls and so it is falling free. The only acting force in this region is the gravitational force and so the trajectory in this case can be obtained by solving the ODE

$$\frac{d}{dt}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = g \begin{pmatrix} 0 \\ -1 \end{pmatrix}. \tag{2.3}$$

This type of movement is also independent of the number of the layer.
We solve the ODE's for the three different regions. The initial condition for Equation 2.1 is given by the rotation speed of the mill. The initial condition for Equation 2.2 is given by the velocity of a ball at the critical angle which is the velocity at the end of region ①. And finally the initial condition for equation 2.3 is given by the velocity at the end of the chain movement, given by Equation 2.2.

> **Technical realization:** We use the MATLAB-file ballmill.m to calculate the trajectories of the different layers in our mill. The inputs are the number of layers to be calculated, the information whether there is liquid inside or not (here: 0 in case of no liquid), the volume of liquid inside in liters (has no influence if no liquid is inside), the speed of the mill in rotations per minute and the total mass of balls in kilograms in the mill. The program solves the ODE's derived for the different regions and plots the trajectories into the mill depending on the inputs. All numerical integrations are done by a second order explicit Runge-Kutta method

The problem occurring now is that we do not know up to this point until to which point we have to calculate the trajectories because we have no information about the surface of the ball filling.

2.3 Surface structure of the ball filling

In this section we want to calculate the surface structure of the ball filling to get an impression where the trajectories of the flying balls end.
To do so we make several assumptions:

- we consider the surface to be smooth, because the balls are small
- there are no balls flying around
- the surface is in a steady state, which means that the balls move but the surface stays constant

Under these assumptions we can consider the forces acting on a point mass on the surface and then we can derive a formula for the surface structure by a force equilibrium ansatz.

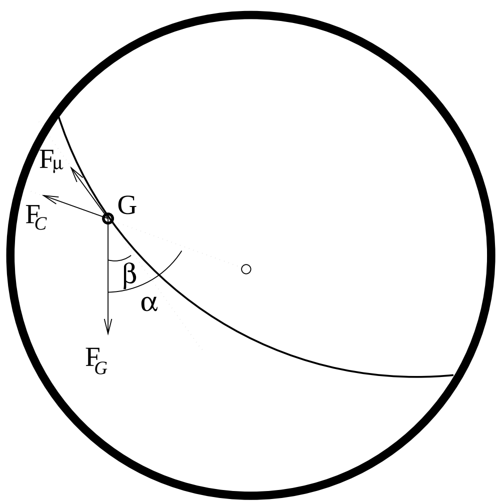


Figure 2.7: Forces acting on a mass point on the surface

We consider a mass point G on the surface. The forces acting on this point are

- gravitational force $F_g = mg$
- centrifugal force $F_C = ml\omega^2$ where l is the distance from the origin to G
- frictional force F_μ

Then we can decompose the forces into two forces, one tangential to the surface, F_t, and one perpendicular to the surface, F_n. We obtain

$$F_t = mg\cos(\beta) - ml\omega^2\cos(\alpha-\beta) - F_\mu,$$
$$F_n = ml\omega^2\sin(\alpha-\beta) + mg\sin(\beta),$$
$$F_\mu = \mu F_n,$$

where μ is the coefficient of friction between the balls.
For the mass point to be in an equilibrium state we need the tangential force being equal to zero. So we get

$$\mu\big(ml\omega^2\sin(\alpha-\beta) + mg\sin(\beta)\big) = mg\cos(\beta) - ml\omega^2\cos(\alpha-\beta)$$

Using

$$y = \cos(\alpha),$$
$$x = -l\sin(\alpha),$$
$$\frac{dx}{dy} = -\cot(\beta),$$

we obtain the ordinary differential equation

$$\frac{dx}{dy}[\mu x - (y - \frac{g}{\omega^2})] = \mu[y - \frac{g}{\omega^2}] + x.$$

By substituting $t = y - \frac{g}{\omega^2}$ one can simplify the ODE to

$$\frac{dt}{dx} = \frac{x + \mu t}{\mu x - t}.$$

We can solve this first order homogeneous ODE analytically and get

$$x^2 + t^2 = A^2 e^{2\mu \arctan\frac{t}{x}},$$

where A is a constant. Resubstituting gives

$$x^2 + (y - \frac{g}{\omega^2})^2 = A^2 e^{2\mu \arctan\frac{y-\frac{g}{\omega^2}}{x}}. \tag{2.4}$$

Equation 2.4 is the implicit formula for the surface structure. In this equation we have two parameters: A and μ. μ is depending on the friction between the balls. With A constant, varying μ influences the steepness of the surface (see Figure 2.8).
We choose a value $\mu = 0.65$ (taken from literature). A is depending on the volume under the surface of balls. With μ constant the height of the surface changes (see Figure 2.9).
In our case the volume is given by the number of balls. According to this given number of balls we can calculate the volume and by this the constant A.

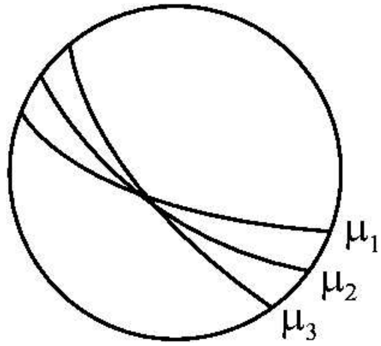


Figure 2.8: Steepness of the surface depending on μ, $\mu_1 < \mu_2 < \mu_3$

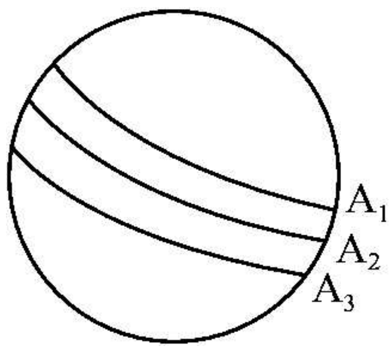


Figure 2.9: The surface depending on the volume of the filling, $A_1 > A_2 > A_3$

2.4 Combining the results of Section 2.2 and 2.3

We know that the balls are lifted up above the surface in the left region. But by calculating the surface of the ball filling we get better approximations for the endpoints of the trajectories of the balls.
But now another problem has to be determined: the surface structure is calculated with all the balls in our cylinder but there is a certain amount of balls lifted up above the surface. So we have to reduce the number of balls under the surface by the amount of balls above the surface and by this we get an improved surface structure.

> **Technical realization:** In the MATLAB-file we calculate the surface structure for the given total number of balls by solving the implicit formula derived in this chapter by a Newton-iteration to get explicit values. Then we calculate the volume of the balls above the surface and weight that volume with a factor depending on the volume itself (the packing rate of flying balls is the lower the more balls are flying). Then this weighted volume is subtracted from the total volume of balls. Then we restart the same calculations, but with a new volume of balls. This iteration is performed four times (we do not perform more iteration steps, because each step requires many calculations

and further iteration steps do not change the result much). To calculate the ball volumes we use the trapezoid quadrature rule, which is good here, because the surface has a low curvature. Now we can calculate the trajectories of the balls up to the calculated surface.

And so we finally achieve the surface structure for the correct number of balls under the surface and with these results we are now able to calculate the energy of the balls when they hit the surface.

2.5 Results, conclusions and restrictions

As a result of the previous calculations we now can read from the program the velocities of a ball in each layer when it hits the surface and by this we can calculate the energies of the balls.

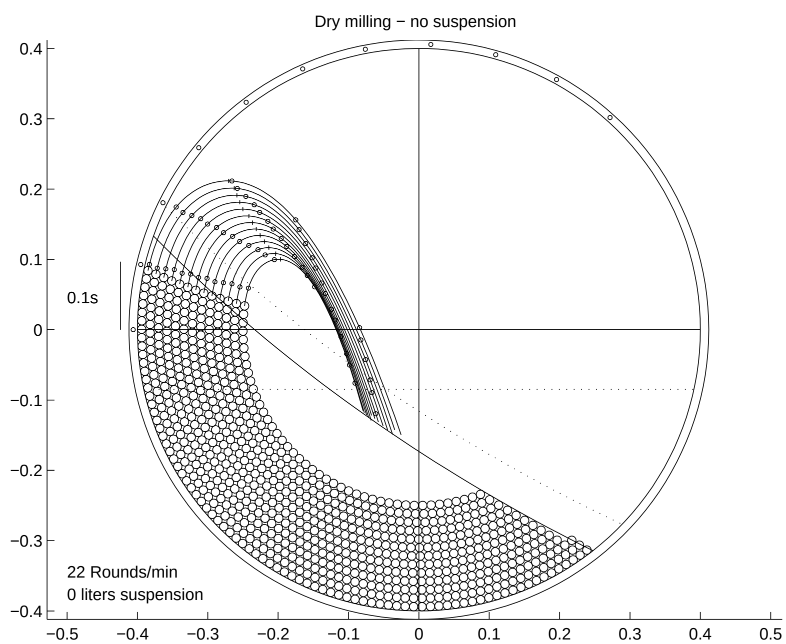


Figure 2.10: Trajectories of the balls

The dotted straight line is the filling height of the balls without rotation and the other dotted line is the surface of the balls in the rotated mill without volume correction. The small and thin line, which is close to the top of each trajectory, shows the position where the chain movement ends (region ② in Figure 2.3) and the free fall begins (region ③ in Figure 2.3).
For a ball in the first layer:

velocity: 2.84 $m \cdot s^{-1}$
energy: 52.9 mJ

Compared to the speed of the mill at the wall, which is 0.9 ms^{-1}, this is a quite high value. But of course the calculated velocity is an upper bound because of the rearrangement of the balls: the calculated curves for the different layers come closer than twice the radius of a ball, which is the minimal distance of two balls. So the layers will rearrange because the collision angle is very small (they fly nearly parallel). And by this rearrangement the energy is distributed to the inner layers that do not have so much energy compared to the first layer.
But these calculations are based on several assumptions in which errors are included:

1. In reality our mill is of cause 3D and not 2D. A cut through this 3D mill would not consist of equal sized discs and we would have a certain movement in the direction of the main axis.

2. We considered a perfect layer structure but this is not the case in reality: in the first layer we have almost the assumed perfect layer structure, only in the region of the steel bars there are small disturbances. But beginning with the second layer the balls will fit to the gaps of the outer layer and by this the layer structure is getting worse and worse when we move to the center of the cylinder. Further we assumed that the angular velocity of the different layers is the same than the angular velocity of the mill. This is in general not the case since due to the non perfect layer structure the balls are sliding down from each other and so the angular velocity of the inner layers change.

3. The "steady state" assumption is generally true but there may be some changes because of the gaps between layers and the periodical influence of the steel bars at the cylinder wall. But since there are enough balls following from behind and since there are no such gaps in the other layers, we assume that this has not a big influence on the movement.

4. We neglect the self-rotation of the balls itself because in the cylinder the balls will interact and rotate against each other and so we will not get very fast rotation speeds.

In this chapter we were able to give an approximate upper bound for the energy of a ball from the calculation of the energy of a ball in the first layer. Further we were able to get approximate values for the energies of the balls in each other layer.
Now we can go on with our calculations and include the influence of the suspension inside.

Chapter 3

Wet Milling with Liquid at Rest

3.1 Basic ideas and assumption

Now we can consider the influence of the suspension. Ball milling with suspension is called wet milling in literature (see Figure 3.1). For simplification in our first model we are going to consider the suspension at rest. Of course in reality, this is not true. But by this assumption we can get important estimates. Further we assume the surface of the suspension to be a horizontal line.

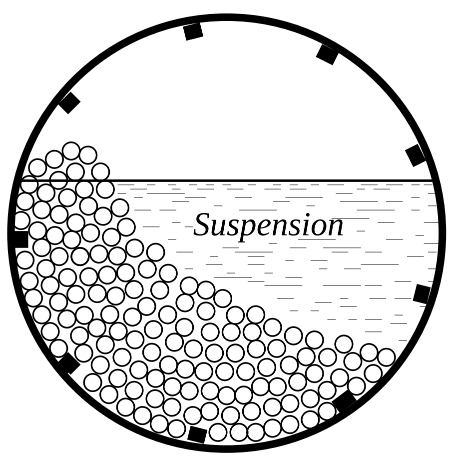


Figure 3.1: Wet milling with suspension at rest

Considering the balls inside the suspension, beside gravitational force and centrifugal force there are two additional forces: buoyancy force and Stokes force (see Figure 3.2). These forces are acting when the balls are under the surface line of the suspension. If the balls are leaving the suspension in this model, these two additional forces are not acting any more.
The different forces acting on a ball are :

F_C : centrifugal force (in radial direction)

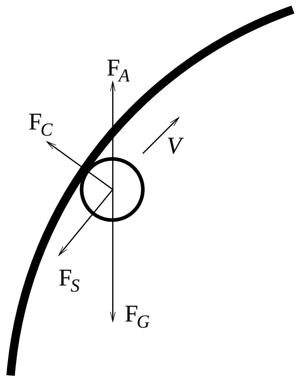


Figure 3.2: Forces acting on a ball in liquid

F_G : gravitational force
F_A : buoyancy force (opposite direction of F_G)
F_S : Stokes force (opposite direction of velocity V)
V : velocity of ball (perpendicular to F_C)

3.2 Movement of balls

Now we consider the motion of balls with liquid at rest. As considered in dry milling (see Figure 2.3), we consider the movement of balls in three different regions.

In region ① the movement of the balls is nearly the same as in dry milling but the critical angle is changed because of the buoyancy force. In region ② the movement of the balls is nearly the same as in dry milling again because the influence of the Stokes force is taken by the following balls and so by the steel bars at the end. The influence of the buoyancy force will change the position of the balls losing contact with the chain. In region ③ the balls start losing contact with the balls behind and begin to fall down freely under the influence of Stokes force, buoyancy force and gravitational force.

The critical angles for the k-th layer can be determined like in dry milling, only the buoyancy force has to be included. This critical angles α_{ck} are now given by the following formula :

$$\alpha_{ck} = \arcsin \frac{m(R-(2k-1)r)\omega^2}{(m-\bar{m})\cdot g}$$

where $\bar{m}$ is the mass of the volume of a ball replaced by suspension.
To describe the movement of balls after losing contact with the cylinder wall, consider the forces acting on a ball in region ②:

F_G : gravitational force

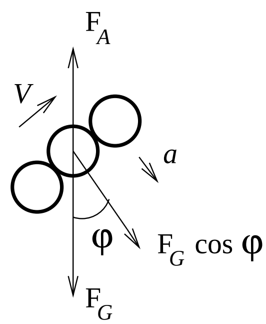


Figure 3.3:

F_A : buoyancy force
a : acceleration of a ball towards the center of the mill.
V : velocity of a ball perpendicular to a.

Since a is perpendicular to V it follows that $|V|$ is constant. But a is not a constant because $a = a(\varphi)$. By this we obtain the ODE:

$$\frac{d}{dt}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \left(\frac{(m-\bar{m})\cdot g}{m}\right)\begin{pmatrix} \frac{V_1 V_2}{V_1^2+V_2^2} \\ \frac{-V_1^2}{V_1^2+V_2^2} \end{pmatrix} \tag{3.1}$$

Considering the forces acting on a ball in region ③ shown in Figure 3.3, we obtain the following equation :

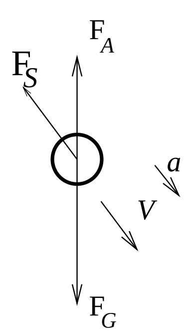


Figure 3.4: Forces acting in the movement in a chain

$$\frac{d}{dt}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \frac{1}{m}\begin{pmatrix} -F_S\frac{V_1}{\sqrt{V_1^2+V_2^2}} \\ (m-\bar{m})\cdot g - F_S\frac{V_2}{\sqrt{V_1^2+V_2^2}} \end{pmatrix} \tag{3.2}$$

with $F_S = ...$

Technical realization: Inside the suspension, buoyancy force and Stokes force are acting, above the suspension surface not. To distinguish these cases, in each step the program compares the distance from the balls and the surface to the x-axis. If the distance of the balls from the x-axis is bigger than that of the surface, then the program runs without considering Stokes and buoyancy forces. But here a new problem appears: if balls cross the suspensions surface, this means a discontinuity of the acting force on a ball in our calculations. Nevertheless this will not produce any important numerical errors, because our maximal step size is small enough.
In spite of this the effect of balls leaving and falling back onto the suspension's surface will be stronger than just a discontinuity of the force. So curves which leave the suspension must be considered carefully.

3.3 Surface structure of the balls

To calculate the surface structure in wet milling we make the same force equilibrium ansatz as in dry milling, we only have to include the buoyancy force.

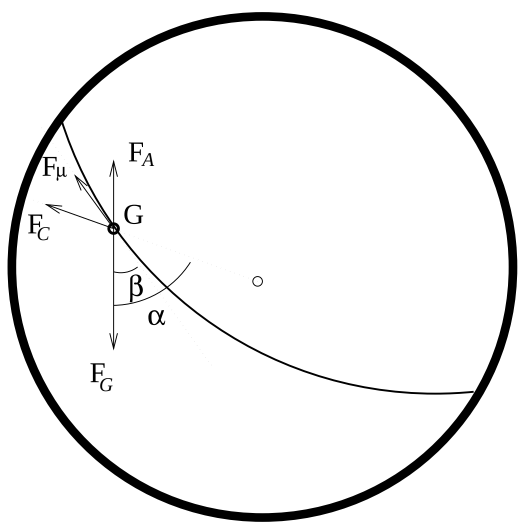


Figure 3.5: Forces acting on a mass point at the surface in suspension

Considering the additional buoyancy force we get the formula for the surface structure in wet milling as follows:

$$x^2 + (y - \frac{(m - \bar{m}) \cdot g}{m\omega^2})^2 = A^2 e^{2\mu \arctan(\frac{y - \frac{(m-\bar{m})\cdot g}{m\omega^2}}{x})} \tag{3.3}$$

This is nearly the same implicit equation as in dry milling (2.4). Normally the frictional coefficient μ has to be modified because of the influence of the suspension, but since we do not have any measurements or values from literature we keep the same value as in dry milling.

3.4 Result and conclusion

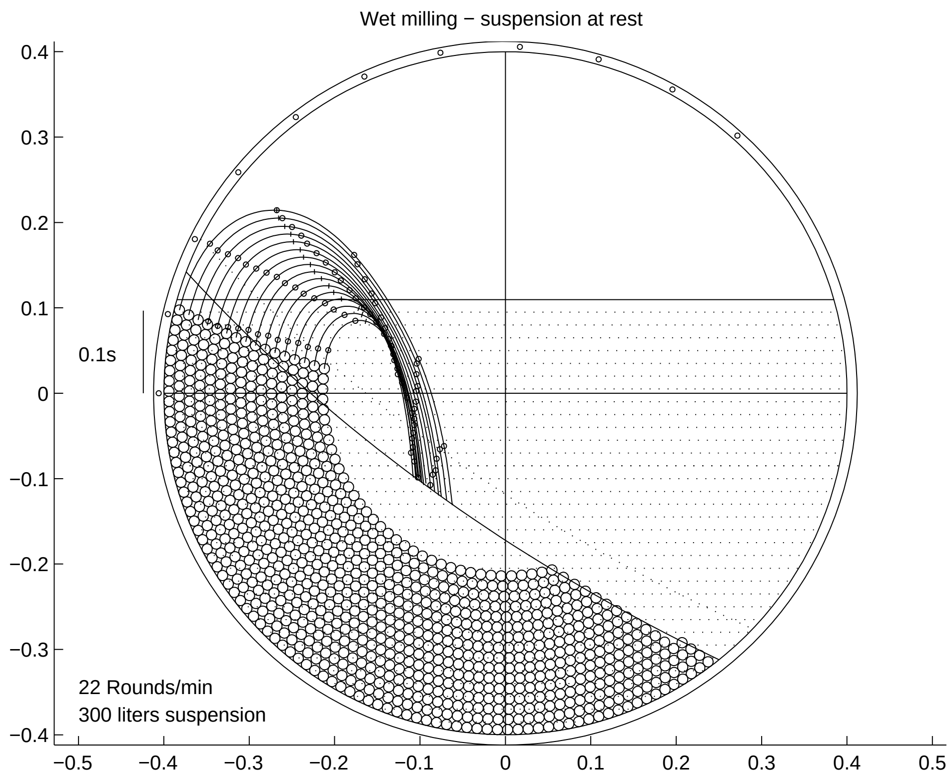
We achieved the following results for the milling with suspension at rest and we compare it with dry milling.

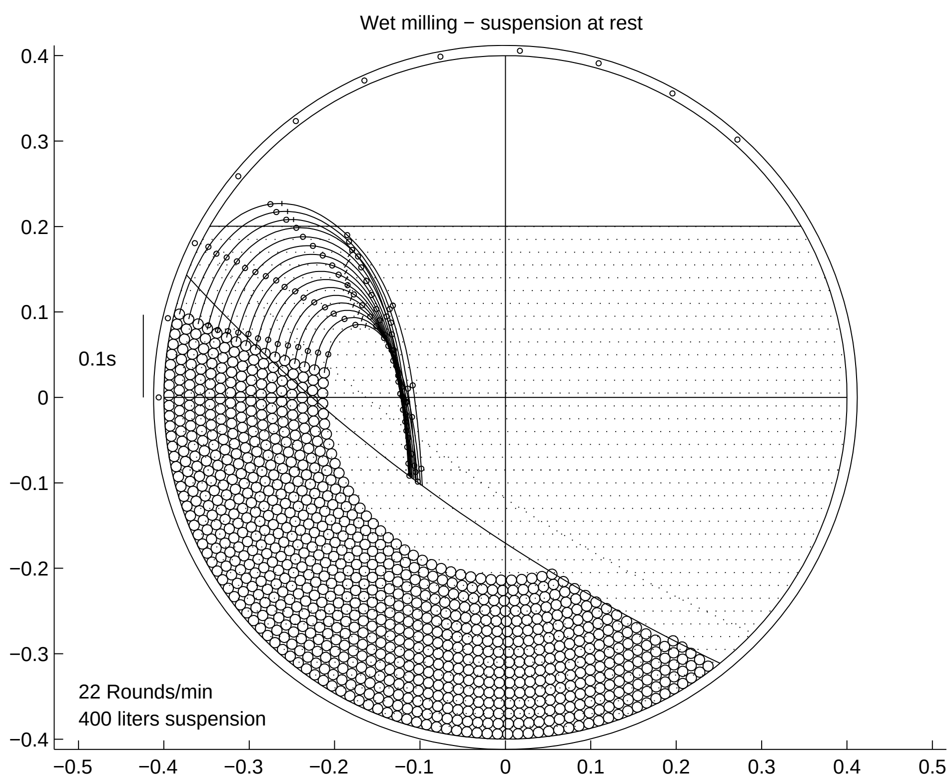
	Critical Angle	Velocity	energy
Dry milling	10°	2.7 ms^{-1}	48 mJ
Wet milling			
300 l of suspension	12°	0.95 ms^{-1}	5.9 mJ
400 l of suspension	12°	0.95 ms^{-1}	5.9 mJ

According to our result, we can conclude :

- The influence of the Stokes force is rather big in region ③ in Figure 2.3 as we compare the resulting velocities when hitting the surface.
- The different filling heights change the form of the curves of the balls, but the velocities of the balls when they hit the surface of balls do not differ much.
- Like in dry milling the layers will rearrange in wet milling, too, and so we get an upper bound for the energies and the velocities.

Ball curves for resting suspension





Chapter 4

Wet Milling with Moving Liquid

The suspension will be moved by the rotation of the cylinder just like the balls. Further the movement of the suspension influences the movement of the balls and vice versa. In this chapter a moving suspension and its influence on the balls' movement will be modeled. On the other hand the balls' influence on the suspension will be neglected.

4.1 The velocity field of the suspension

First the movement of the suspension without any balls has to be calculated. The main difference to the balls' movement is that the suspension is a continuous medium, while balls are discrete objects. So the balls' movement can be described by ordinary differential equations, while the suspension's movement is the solution of the Navier-Stokes-equation, a partial differential equation.
If we have the cylinder filled with a certain volume of suspension, we are interested in two outputs:

- The velocity field of the suspension
- The surface structure of the suspension

Again it is reasonable to consider only a 2D-model, for the same reasons as with the balls' movement.

Calculating the surface structure is a so called "free boundary problem", which is quite complicated. So we first consider the easier case, when there is no surface, i.e. the whole cylinder is filled with liquid.

We used the program FLUENT, a numerical solver for Navier-Stokes-equations, and achieved the following result.
In Figure 4.1 the angular velocity field is plotted. The radial velocities are almost zero everywhere, so we have nearly a perfect rotation process around the main-axis of the cylinder, respectively the mid point of the circle. So it can be concluded that gravitation has nearly no influence on the movement of the suspension (otherwise the fix point of

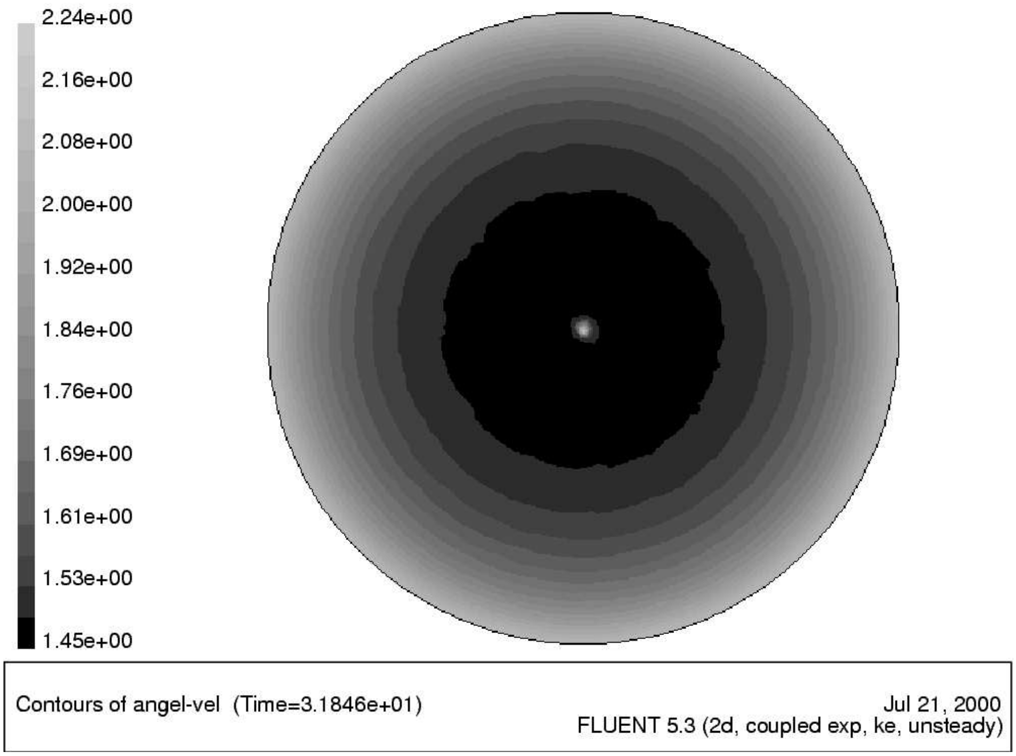


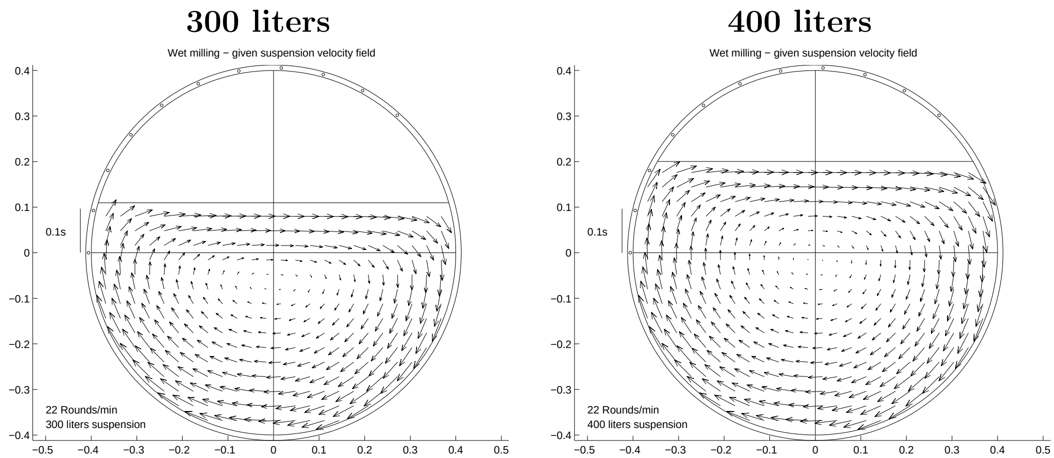
Figure 4.1: The velocity field in the cylinder computed with FLUENT

the rotation would be lower than the circle's mid point).
Furthermore the suspension's velocity at the wall is the same as the cylinder's velocity, i.e. we have no slip at the wall. Towards the middle of the cylinder the angular velocity of the suspension decreases, i.e. the outer liquid layers rotate faster than the inner layers. From this fluent output the decrease of angular velocity towards the middle can be estimated and can be used for the next step. (One can see that at a distance of $\frac{1}{2}R$ with R the cylinder's radius the angular velocity is about 75% of the cylinder's angular velocity.)

In our problem are surfaces corresponding to the filling heights of 300 l or 400 l of suspension, so reasonable velocity fields for these cases have to be achieved, too. Of cause one cannot just take 300 l respectively 400 l of suspension and remove the balls, because then the filling heights would be much too low. The considerations must be performed for the movement of such an amount of suspension, such that the same filling heights are achieved as if there were balls. Hence the volume of the balls (207 l) has to be substituted by the same volume of suspension to achieve the same filling heights. So the calculations have to be performed for suspension volumes of 507 l and 607 l.

Unfortunately, we were not able to perform this free boundary calculations because of the numerical instability. So the following velocity fields were constructed by our expectations to reality.

These are the constructed velocity fields for 300 l and 400 l.



Bigger pictures are attached at the end of this section.

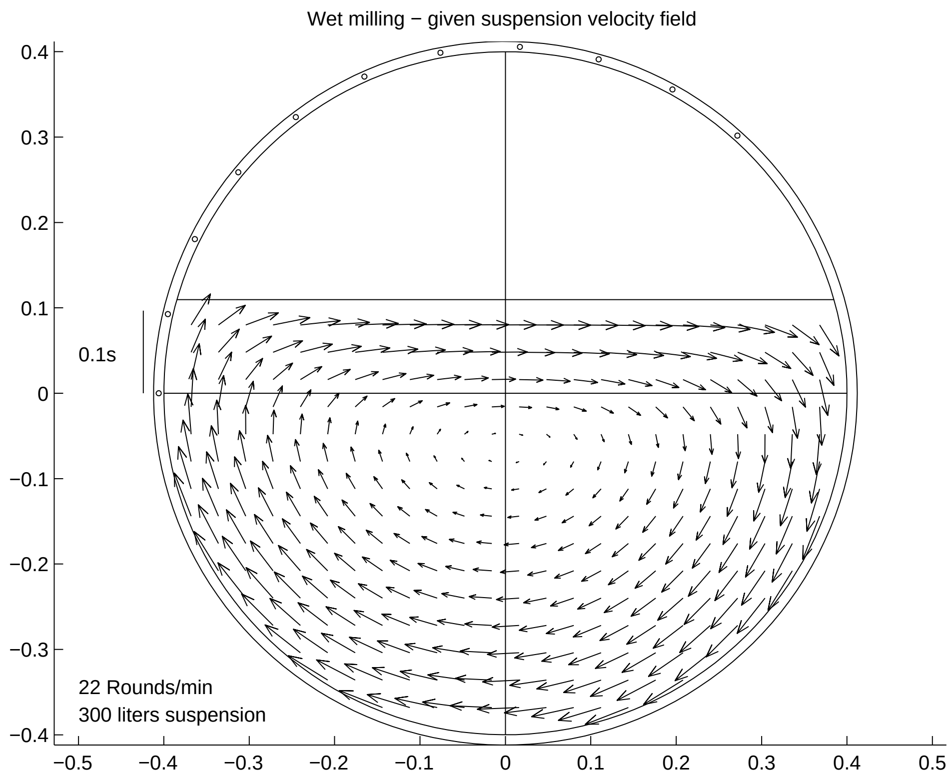
These velocity fields are constructed by starting with the velocity fields of the calculated case with the whole cylinder filled (Fluent output). Then a horizontal surface line is drawn at the correct filling height for the given volume. The suspension velocity vector near this line must be parallel to it. By conservation arguments one can conclude that the velocity at the surface must be the same as the velocity of the cylinder wall. By a weight function which is 1 at the surface line and strongly decreasing with increasing distance we combine the two vector fields.

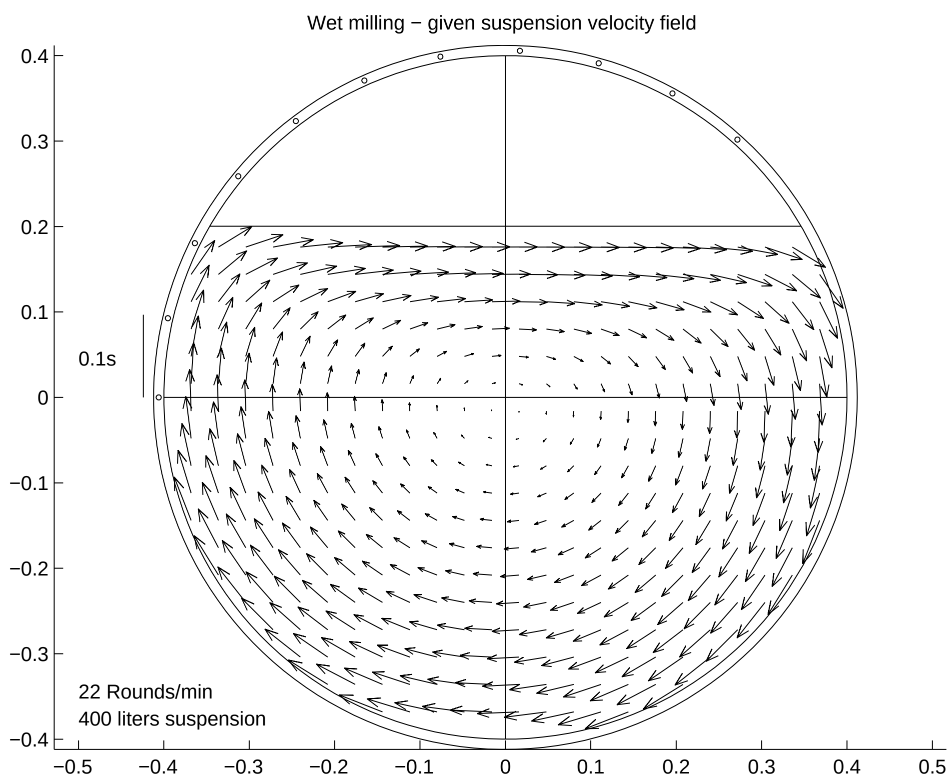
> **Technical realization:** In the Matlab-program the velocity field is saved in a cartesian 26×26-grid. So arbitrary velocity fields can easily be used for the ball movement calculations. The velocity vector at an arbitrary coordinate inside the mill and under the suspension surface is achieved by bilinear interpolation of the four surrounding gridpoint vectors.

In reality the surface of the suspension will not be a horizontal line, but to obtain any numerical estimate it was chosen this way. At least from the given viscosity it can be estimated that its slope should be much smaller than the slope of the balls' surface. This is certainly one point where the model should be improved.

These constructed velocity fields will be used for the next step.

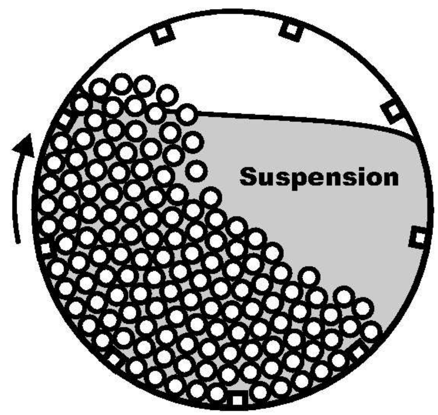
Suspension velocity fields





4.2 The balls' movement under the suspensions' influence

Now the suspension's influence on the balls' movement is modeled. Compared to the case with suspension at rest we obtain an other value for the Stokes force, while the three other forces acting on one ball (gravitation, centrifugal force, buoyancy) are of the same size. The Stokes force acting on one ball points in the opposite direction of the relative velocity between ball and suspension. So the form of the curves the balls describe can be influenced strongly by the moving suspension. Again it should be remarked that here the balls' influence to the suspension is not modeled.

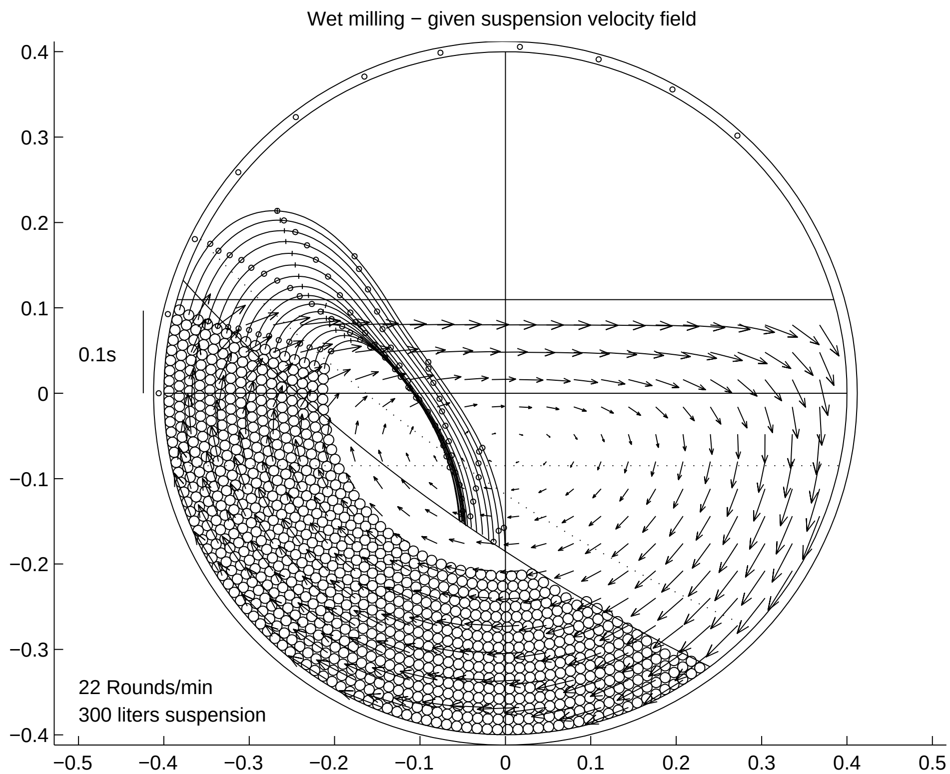


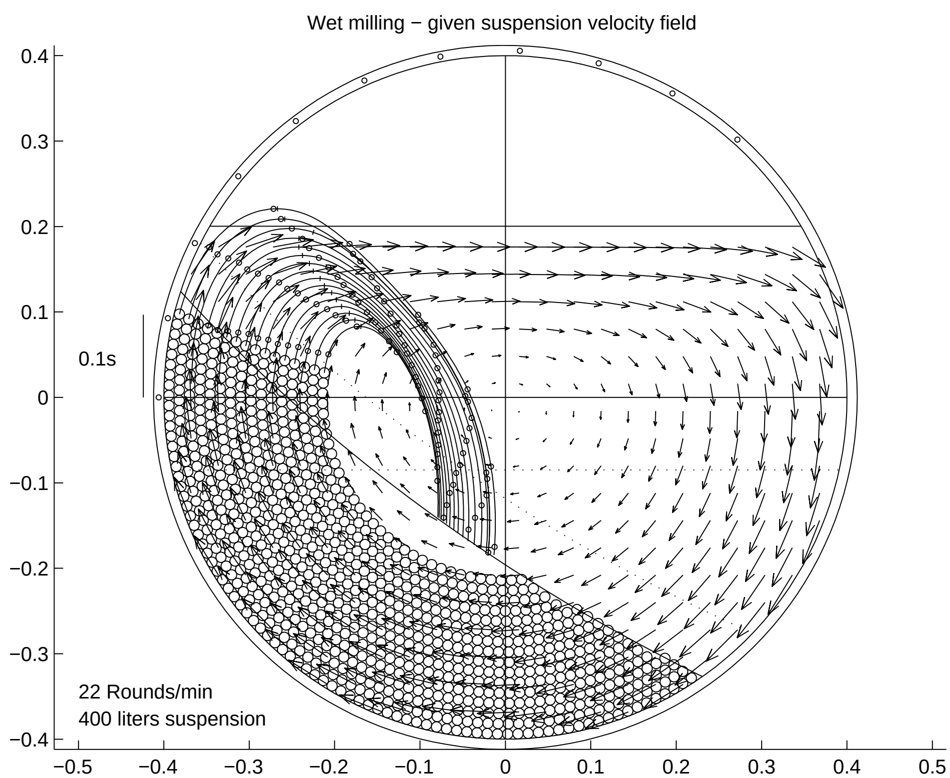
Technical realization: First it is checked whether a given ball is inside the suspension or above. In the second case, buoyancy and Stokes force are set to zero, while in the first case the suspension's velocity at the ball's position is calculated by bilinear interpolation. Then the Stokes force is calculated from the relative velocity of ball and suspension. Gravitation and centrifugal force (if active) are added. The integrations are performed like in the previous cases.

The maximum kinetic energies of balls hitting the surface are:

Filling volume	Velocity	Kinetic energy
300 liters	0.951 ms^{-1}	5.93 mJ
400 liters	0.955 ms^{-1}	5.98 mJ

Ball curves for moving suspension





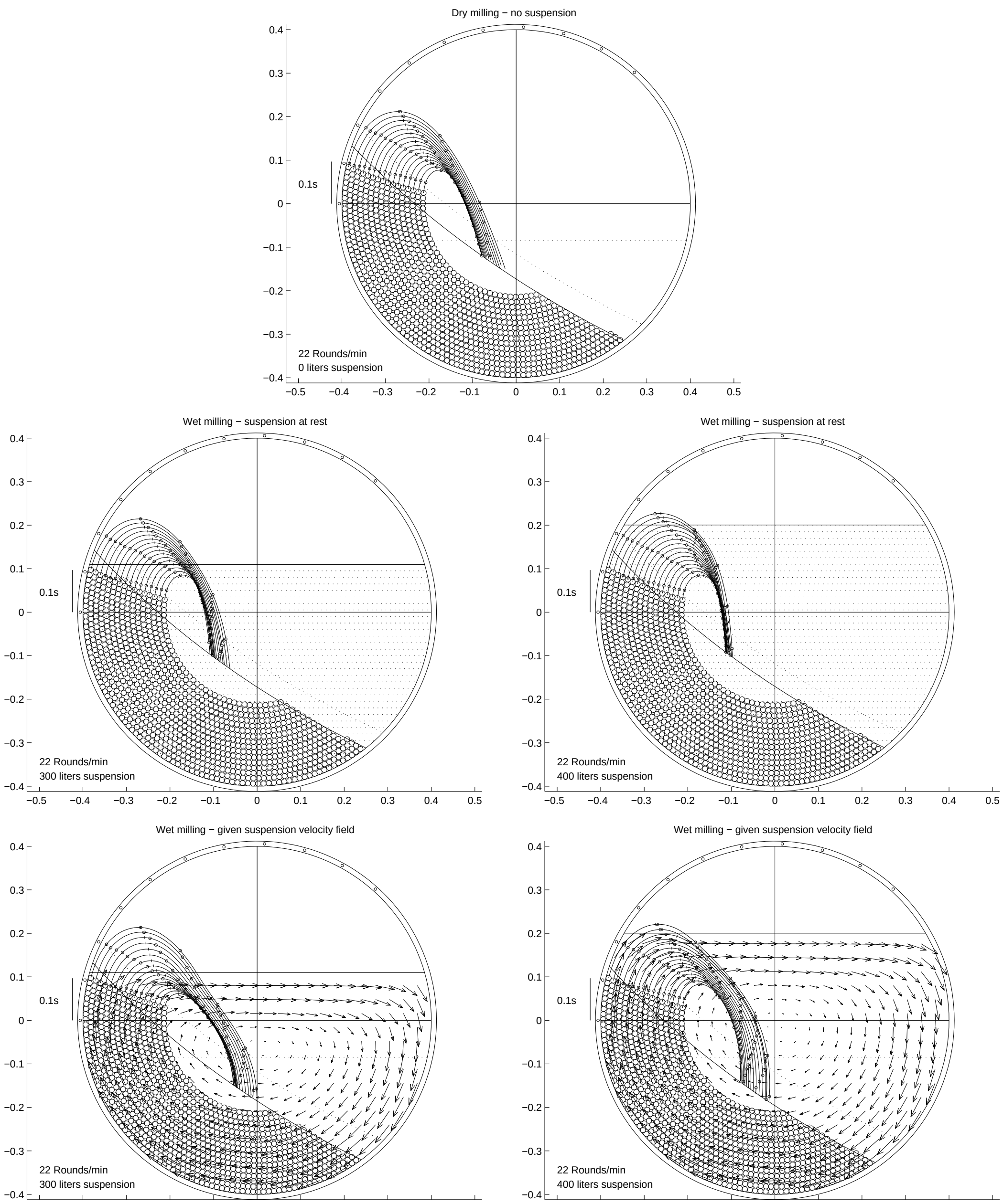
Chapter 5

Conclusions and Outlook

5.1 Comparing the three models

The maximum energies of balls hitting their surface in the three considered cases are:

Filling volume	300 liters		400 liters	
Dry milling	2.84 ms^{-1}	52.9 mJ	2.84 ms^{-1}	52.9 mJ
Resting suspension	0.993 ms^{-1}	6.47 mJ	0.987 ms^{-1}	6.39 mJ
Moving suspension	0.951 ms^{-1}	5.93 mJ	0.955 ms^{-1}	5.98 mJ



The following conclusions can be drawn:

- **Dry milling vs. wet milling**
  - The velocities respectively the energies of balls hitting the surface formed by the other balls are much lower in both wet milling cases compared to the dry milling case. The reason is that the suspension drags the balls' free falling by the Stokes force. Here the results for a moving suspension are quite close to those with a resting suspension.

- **Resting suspension vs. moving suspension**
  - The main difference between moving and resting suspension is that a moving suspension takes the balls much higher and longer over the surface than a resting one. Especially near the surface the suspension drags balls towards the right side of the cylinder.
  - As an important result the number of balls forming the surface is much lower, if there are more balls being lifted up. Therefore there will be even more flying balls, and even a hollow between flying balls and balls forming the surface can appear.
  - Rearrangement, which reduces the velocities for the balls in the outer layers by the transport of energy to the inner layers, is stronger the closer the layers come. Therefore it will take place mostly in the wet milling case with resting suspension, while the effect will be lower in the case of moving suspension.
  - This effect is much stronger than the fact that the calculated energies in the case of moving suspension are a little bit lower than in the resting case. The reason for this at a first glance surprising data is that balls hit the surface at a point where the suspension is moving upwards again. Of cause in this region the suspension velocity field will be changed totally by the balls' movement. Therefore the energy difference between the two wet milling cases seems to be negligible in reality.
- 300 l **vs.** 400 l
  - In the dry milling case there is no difference between the two cases, because there is no suspension at all.
  - In both wet milling cases the final energies do not differ very much (compare the table at the beginning of Chapter 5).
  - Especially in the case of moving suspension balls are lifted up higher in the case of 400 l. Therefore there are more flying balls if more suspension is taken and as a result – like in the previous consideration – less rearrangement. So with 300 l of suspension there will be less flying balls. But the difference is not very big.
- **Number of balls**
  - It is obvious that the number of flying balls and the energies of balls hitting their surface can easily be reduced by just taking more balls. Firstly the surface will be lifted up, secondly a hollow will be prevented and thirdly the effect of rearrangement will be increased.

5.2 Conclusions and results

From our model we were able to derive answers to the questions posed by the company.

Question: What is the movement of the balls when the mill is rotating?

Answer: The balls will be lifted up in the left region and fall back onto their own surface like in Figure 5.1.

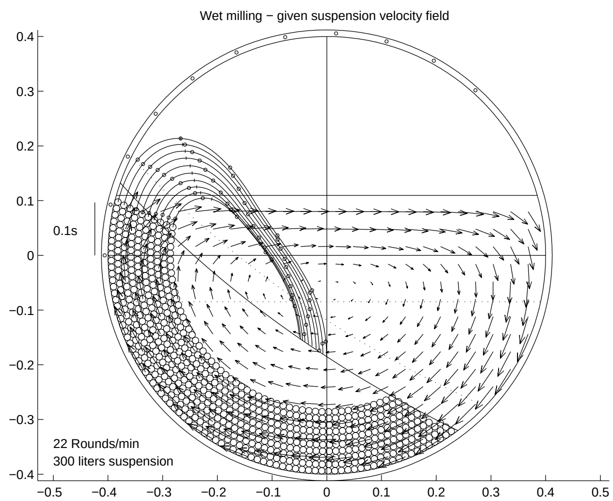


Figure 5.1: Movement of the balls for 300 l suspension

Question: Which surface do the balls form?
How will the surface of the suspension look like?

Answer: The ball surface will look like in the curve in 5.1 where the trajectories of the balls end. Further we expect the surface of the suspension to be close to a line with a small curvature near the walls but we were not able to solve this free boundary problem.

Question: What is the energy of the balls when they hit the ball surface?

Answer: The suspension reduces the velocity of the falling balls a lot. So the energy for each ball collision will probably not be much higher than 6 mJ. If we take the rearrangement of the different layers into account the maximal energy will be even lower.

Question: What is the dependence on the volume of the suspension?

Answer: The volume of the suspension does not influence the energy of the balls hitting the surface very much. With more suspension the amount of balls over the balls surface is increased. Further with only 300 l of suspension the balls can leave the suspension (see Figure 5.1) and with 400 l this will not happen if we consider the suspension to be taken with the balls a little bit (see Figure 5.2).

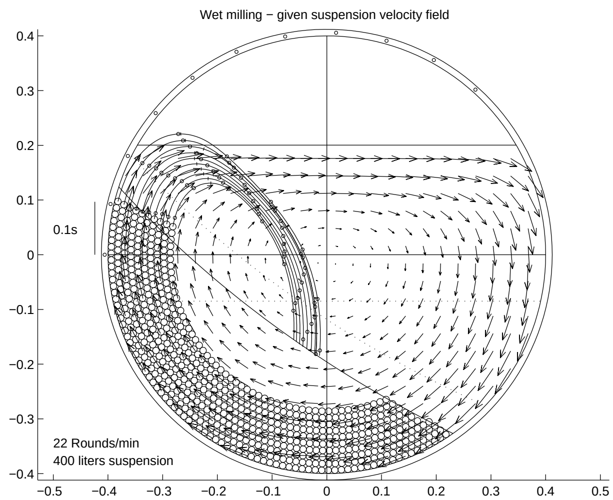


Figure 5.2: Movement of the balls for 400l suspension

5.3 Parameter discussion

In our model we have at least three important parameters:

1. Rotation speed:
   In our model we can change the rotation speed of the mill to get different results e.g. different trajectories or hitting energies. But the engines are optimized for one certain rotation speed and so this parameter cannot be changed.

2. Volume of suspension:
   In the cases considered in our model (300 l or 400 l) this parameter seems not to be important for the energy of balls hitting the surface. From the model we can observe that more suspension means that more balls are lifted up above the surface calculated for the ball filling and with fewer suspension the balls are lifted out of the suspension.

3. Volume of balls:
   An increase of the number of balls would reduce the number of flying balls. But the problem that can occur is that the filling could become too heavy.

As a result of this we can conclude that for a lower kinetic energy of the balls when they hit the surface it is better to use more balls and more suspension.
But in this result the efficiency of the mixing process is not taken into account.

5.4 Outlook

For further research several improvements of the model can be performed:

1. The influence of the different layers to each other (rearrangement) could be described.
2. The surface of the suspension could be calculated by solving the free boundary problem. Then in the model the correct surface can be used.
3. The influence of the balls to the suspension could be modeled.
4. The whole movement of the balls and the suspension at once could be modeled.
5. The process could be modeled as an n-body problem.
6. The mixing process could be taken into account in order to increase the efficiency of the ball mill.

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