

The Little Prince and his Pendulum

Math 51, Winter 2005

The Little Prince lives on a tiny, lonely planet. To amuse himself, he makes a simple pendulum from a rubber band and a rock. Someone once told the Prince that the oscillation time of a pendulum (the time it takes to go through one full period) is given by the formula

$$T = 2\pi\sqrt{L/g},$$

where L is the length of the pendulum and g is the gravitational acceleration.

As the Little Prince lifts the pendulum higher in the air, the gravity gets smaller, so the rubber band also gets shorter. (Since his planet is so small, both effects are quite noticeable.) In terms of the height off the surface of the planet,

$$\begin{aligned}L(h) &= \frac{1}{4} + \frac{1}{4h} \quad m, \\g(h) &= \frac{100}{(h+2)^2} \quad m/s^2.\end{aligned}$$

1. Use the multivariable chain rule to compute $\frac{dT}{dh}$, the rate of change of the oscillation time, when h is 1 meter.

2. Substitute the formulas for $L(h)$ and $g(h)$ into the equation for T and compute the derivative directly. Do you get the same answer?