

# Math 9500 presentation ideas

Fall 2022

Below are some ideas for presentation topics. This is not meant to be an exhaustive list! If you find a topic from outside this list, by all means pursue it.

Whether you choose a topic from the list, or outside it, I ask that you come talk to me about your topic the week of November 1–4.

**1.** Research and report on some portion of the homeomorphism problem for 3–manifolds. Any one of the following is an interesting sub-case:

- The  $S^3$  recognition problem (Rubinstein, Thompson)
- The lens space recognition problem (Lackenby–Schleimer)
- The unknot recognition problem (lots and lots of approaches here – pick one or two)
- Distinguishing hyperbolic 3–manifolds by their fundamental groups (Sela) or by computing the hyperbolic structure (Casson, Manning, Scott–Short)
- Computational complexity (embedding known  $NP$ –hard questions into 3–manifold problems)

**2.** Constructions of 3–manifolds. There are several interesting ways to build them, surveyed in Chapter 11 of Martelli. We have barely touched on these. Choose one of:

- Heegaard splittings
- Dehn surgery on knots and links
- Surface bundles over  $S^1$

If you choose one of these topics, you should discuss existence (which 3–manifold be expressed this way?) as well as non-uniqueness (how do you pass between two different Heegaard splittings or surgery descriptions of the same  $M$ ?).

**3.** Pick a type of geometry. Give a reasonably detailed description of it and the 3–manifolds modeled on it.

**4.** In the special case of knot complements, the geometrization theorem is often stated as “every knot in  $S^3$  is either a torus knot, a satellite, or a hyperbolic knot.” Explain how this relates to the “usual” geometrization theorem, stated in terms of the pieces of the JSJ decomposition.

If you feel ambitious, you can add a description of the JSJ decomposition of knot complements (this is well described in a paper by Budney).

Again, there are lots of other possibilities!