Math 9500 Homework 1

Due Thursday, 9/8/22

1. Exercise 9.2.8 in Martelli's book.

2. A *handlebody* is a submanifold of $\mathbb{R}^3$ that is homeomorphic to the regular neighborhood of a connected graph. A handlebody H is said to have *genus* g if its boundary is the surface $S_{g,0}$ of genus g.

For $g \geq 0$ and $b \geq 1$, let $S_{g,b}$ be the orientable surface of genus g with b boundary components. Prove that $S_{g,b} \times I$ is homeomorphic to a handlebody of genus $(2g + b - 1)$. *Hint: consider Euler characteristic.*

3. Let V be a solid torus with meridian m and longitude ℓ. Let W be a solid torus with meridian μ. Suppose that W is glued to V by a homeomorphism $\varphi : \partial W \to \partial V$ such that $\varphi(\mu) = qm + p\ell$. Prove that the resulting space $V \cup_\varphi W$ is homeomorphic to $L(p, q)$.

4. Let $L(p, q)$ and $L(p, q')$ be lens spaces. Prove that if $q' \equiv q^{\pm 1} \mod p$, then $L(p, q)$ has an orientation-preserving homeomorphism to $L(p, q')$.