Math 9120 Homework 2

Due Thursday, 10/13/16

1. Let S be a compact, orientable surface, possibly with marked points. Prove that there is always a collection of simple closed curves and/or arcs that fills S and satisfies the properties (1. – 3.) needed for the Alexander method. Do this without relying on Proposition 3.4 or 3.5 in the book.

2. Let S be a closed surface of genus $g \geq 2$. Find a collection of *separating* closed curves that fills S. You get bonus points if the collection has only two closed curves.

3. Let $S = S_{g,b,p}$ be a surface of genus g, with b boundary components and p punctures. Define the *complexity* $\xi(S) = 3g + (b + p) - 3$, and assume $\xi(S) > 0$. Prove the following facts:

(a) S admits a *pants decomposition*, that is, a collection of simple closed curves $\alpha_1, \ldots, \alpha_k$ cutting S into pairs of pants.

(b) Any pants decomposition has cardinality $k = \xi(S)$.

(c) There are finitely many $Mod(S)$ orbits of pants decompositions. (Can you determine the number for $\xi \leq 2$?)