Math 9100 Homework 3

Due Wednesday, 10/30/19

1. Exercise 3.3.1 in Kassel–Turaev.

2. Prove that for every Artin generator σ_i of B_n, there exists a positive braid $L_i \in B_n^+$ such that $L_i \sigma_i = \Delta_n$.

3. Let α, β be connected, embedded, topologically nontrivial 1–manifolds in a surface S, which intersect transversely. The *bigon criterion* says that if there is an isotopy of α reducing the number of intersection points $\alpha \cap \beta$, then α, β co-bound a bigon. (See Lemma 3.6, Lemma 3.19, and Figure 3.3 in Kassel–Turaev.)

Give a direct proof of the bigon criterion. You may use references, if you wish (for instance, the Farb–Margalit book on Mapping Class Groups is a good source). You may use hyperbolic geometry if it helps. But write out a complete proof.