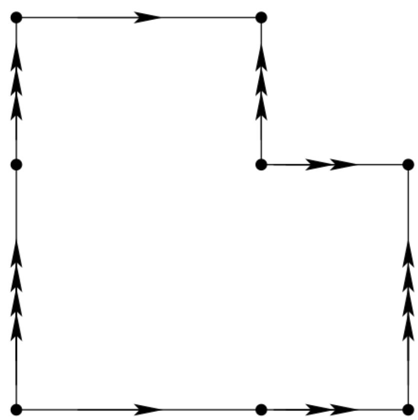
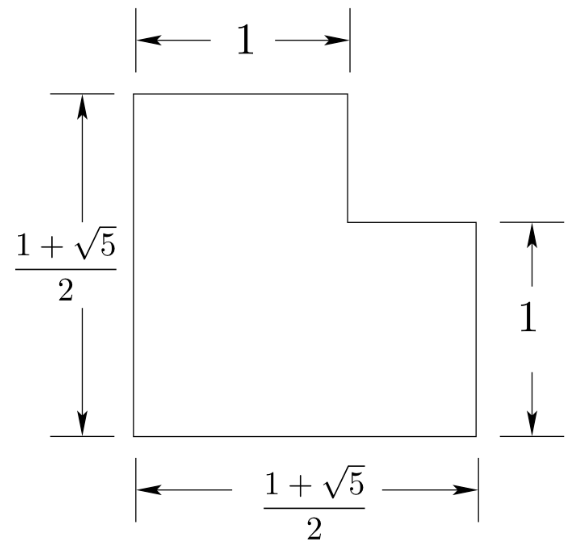
Math 9024 Homework 7

Due Thursday, 4/2/15

Consider the polygon P shown below (bolded dots are vertices). Identify opposite sides by translation. The result is a surface S of genus 2, with all vertices identified to a single point of cone angle 6π.





1. Let $\alpha = \frac{1+\sqrt{5}}{2}$ be the golden mean. Consider the image of P under the linear transformation $\varphi_1 = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}$. Prove that $\varphi_1(P)$ can be cut along straight lines into 4 pieces, with the pieces reassembled by translation to give back P. (In other words, $\varphi_1(P)$ is *scissors congruent* to P.)

2. The above operation on P corresponds to a homeomorphism of the surface S, also denoted φ_1. Prove that this homeomorphism is the composition of two Dehn twists along (disjoint, horizontal) curves in the above polygon.

3. There is a closely analogous homeomorphism $\varphi_2 : S \to S$, associated to the linear map $\varphi_2 = \begin{bmatrix} 1 & 0 \\ \alpha & 1 \end{bmatrix}$. This is the composition of two Dehn twists along (disjoint, vertical) curves in the above polygon.

Let $\varphi = \varphi_1 \circ \varphi_2$. Prove that φ is pseudo-Anosov, by finding its invariant foliations and the dilatation $\lambda(\varphi)$. *Hint:* Consider the eigenspaces of $\varphi_1 \circ \varphi_2$.

Credit: I learned of this example from Christopher Leininger. The pictures are due to him.