Math 9024 Homework 6

Due Thursday, 3/19/15

1. Here is a way to define the Euler characteristic of an orbifold that works in all dimensions. Consider a cell decomposition of Q, in which all points in the interior of a cell σ have the same local group G_σ. (This means that if $\tau \subset \sigma$, then $G_\sigma \subset G_\tau$.) Now, define

$$\chi(Q) = \sum_\sigma \frac{(-1)^{\dim \sigma}}{|G_\sigma|}.$$

In words: every cell counts with a weight, where the weight is the reciprocal of the size of the local group.

Prove that for 2–orbifolds, the above definition agrees with the one given in class.

2. Suppose M is an n–manifold, and H is a finite group acting on M with quotient $Q = M/H$. (You may assume that M has a simplicial structure, and that H acts simplicially. However, H is allowed to have fixed points.) Prove that

$$\chi(M) = |H| \chi(Q).$$

3. Let $K \subset S^3$ be a (p, q) torus knot. Prove that the Seifert fibration on $M = S^3 \setminus N(K)$ extends to all Dehn surgeries on K, with one exception. What is the slope of the excluded surgery?