Math 9024 Homework 3

Due Thursday, 2/12/15

1. Use the area formula for polygons to prove the Gauss–Bonnet Theorem for compact hyperbolic surfaces: $\mathrm{area}(S) = -2\pi\chi(S)$.

2. Let S be a surface with punctures $p_1, \ldots, p_n$, subdivided into ideal triangles. Prove that the completeness equations at the punctures are linearly independent. *Hint:* first, prove this for a favorable choice of triangulation.

3. Let $T \subset \mathbb{H}^3$ be an ideal tetrahedron. Find a $\mathbb{Z}_2 \times \mathbb{Z}_2$ group of symmetries of T. *Hint:* a pair of lines in $\mathbb{H}^3$ with no shared endpoints at infinity has a common perpendicular. See Lemma 2.5.3 in Thurston's book.