Math 9023 Homework 6

Due Friday, 11/16/17

1. Let S be a surface of genus g with $n \geq 1$ punctures. Endow S with an ideal triangulation T, such that each triangle of T is isometric to an ideal triangle in $\mathbb{H}^2$. In class, we proved that the induced hyperbolic metric on S is complete if and only if

$$\sum_{e \text{ an edge into } v} s(e) = 0 \qquad \text{for each puncture } v.$$

a) Prove that as v ranges over the punctures, the above linear equations are independent.

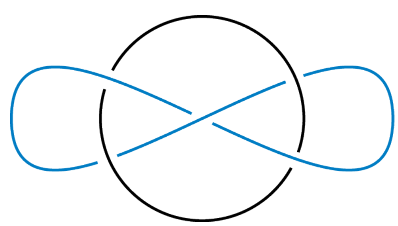
b) Let $\mathrm{Teich}(S)$ be the space of all complete hyperbolic metrics on S. Use the above system of equations to compute the dimension of $\mathrm{Teich}(S)$, in terms of g and n.

2. Do Exercise 3.4 of Purcell's notes, in the special case where $X = \mathbb{H}^2$ and $G = \mathrm{Isom}^+(\mathbb{H}^2)$.

`http://users.monash.edu/~jpurcell/book/Ch03_GeometricStructures.pdf`

3. Let T be an ideal triangulation of a link complement. Prove that T has the same number of edges as tetrahedra. *Hint:* Truncate the tips of the tetrahedra to get a 3–manifold M with boundary consisting of tori. Then, use T to compute the Euler characteristic of ∂M.

4. Let f and g be non-trivial, orientation-preserving isometries of $\mathbb{H}^3$. Prove that f, g commute if and only if they have the same fixed points on $\partial \mathbb{H}^3$.



5. Let $D(K)$ be the standard alternating diagram of the Whitehead link (see above).

a) Work out ideal the polyhedral decomposition of $S^3 \smallsetminus K$ corresponding to $D(K)$, including the step where bigons are collapsed.

b) Add a diagonal in the 4–sided face of the polyhedra (unbounded in the figure above), to subdivide the polyhedra into ideal tetrahedra. Set up the system of gluing and completeness equations for these tetrahedra.

c) For extra credit, solve the system of gluing and completeness equations.