Math 9023 Homework 2

Due Wednesday, 9/19/14

1. Prove that Seifert's algorithm, applied to the standard diagram of a (p, q) torus knot, yields a surface of genus $(p-1)(q-1)/2$.

2. Let S be a Seifert surface of genus g, constructed by applying Seifert's algorithm to a diagram. Prove that $\pi_1(S^3 \setminus S)$ is a free group of rank $2g$.

Note: Not all Seifert surfaces have complements with free fundamental group, so this shows that not every surface can come from Seifert's algorithm.

3. Let K be a (p, q, r) pretzel knot, where p, q, r are all odd multiples of 3. See Figure 1.7 in Lickorish's book for an image. Prove that the genus of K is exactly 1. *Hint:* What is the easiest way to show K is non-trivial?

4. Let K be a non-trivial torus knot. Thus K lies on a torus $T \subset S^3$, with the property $S^3 = V \cup_T W$, where each of V and W is a solid torus. Prove that K is prime, by considering the intersection between a connect-summing sphere and the objects T, V, W.

5. Consider $\mathbb{R}^3 = \mathbb{C} \times \mathbb{R}$, with coordinates $z \in \mathbb{C}$ and $t \in \mathbb{R}$. Let $K \cup L$ be a two-component link in $S^3 = \mathbb{R}^3 \cup \{\infty\}$, where K is the union of the t–axis and the point at ∞. Let $\gamma \subset \mathbb{C}$ be the projection of L to $\mathbb{C} \times \{0\}$. Prove that $lk(K, L)$ is equal to the winding number

$$n(\gamma; 0) = \frac{1}{2\pi i} \int_\gamma \frac{dz}{z}.$$