

Math 9023 Homework 1

Due Wednesday, 9/5/18

1. Prove that the two-component links shown in Figure 1.15 of Prasolov–Sossinsky are all isotopic (and hence are all diagrams of the Whitehead link).
2. Show that any diagram of a knot K can be changed to a diagram of the unknot by changing some of the crossings from “over” to “under”. How many crossing changes are necessary?
3. Let K be an oriented knot in oriented \mathbb{R}^3 . Then the same knot with the opposite orientation is called the *inverse* of K , and denoted rK . If $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is an orientation-reversing homeomorphism (for instance, a reflection), then $f(K)$ is called the *mirror image* of K , and denoted \overline{K} .
Prove that the figure-8 knot is isotopic both to its inverse and to its mirror image.
4. Let K_1, K_2 be knots, and let $K = K_1 \# K_2$. Prove that K is tricolorable if and only if at least one of K_1, K_2 is tricolorable.
5. Do exercise 22(a) on page 55 of Hatcher, which verifies that the Wirtinger presentation is correct. Hatcher’s book can be found online at

<http://www.math.cornell.edu/~hatcher/AT/ATch1.pdf>