Math 9023 Homework 4

Due Thursday, 10/30/14

1. Prove that a non-trivial homotopy class $f \in \pi_1(T^2)$ is represented by a simple closed curve if and only if $f = (p, q)$, where p, q are relatively prime integers.

Note: Prasolov and Sossinsky give a rather complicated proof of this fact in Proposition 14.1. Don't use their argument – instead, use the Change of Coordinates Principle in mapping class groups.

2. Consider the elements $a = (\sigma_1\sigma_2\sigma_3)\sigma_4^2(\sigma_1\sigma_2\sigma_3)^{-1}$ and $b = (\sigma_4\sigma_3\sigma_2)^{-1}\sigma_5^2(\sigma_4\sigma_3\sigma_2)$. What subgroup of B_6 do they generate?

3. Prove that the matrices $A = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 8 \\ 2 & 5 \end{bmatrix}$ generate a free subgroup of $GL(2, \mathbb{R})$.

Hint: Let the matrices act on the upper half-plane by Möbius transformations, and use the ping-pong lemma.

4. Construct a Heegaard splitting of $\mathbb{RP}^3$.

5. Consider the cell complex X obtained by gluing two tetrahedra along their faces, as specified by the picture below. Prove that X has one vertex v, and that the link of this vertex is a torus.

Note: The figure is from Thurston's *Three-Dimensional Geometry and Topology*, and is significant because $X \smallsetminus v$ is homeomorphic to the complement of the figure 8 knot in S^3.

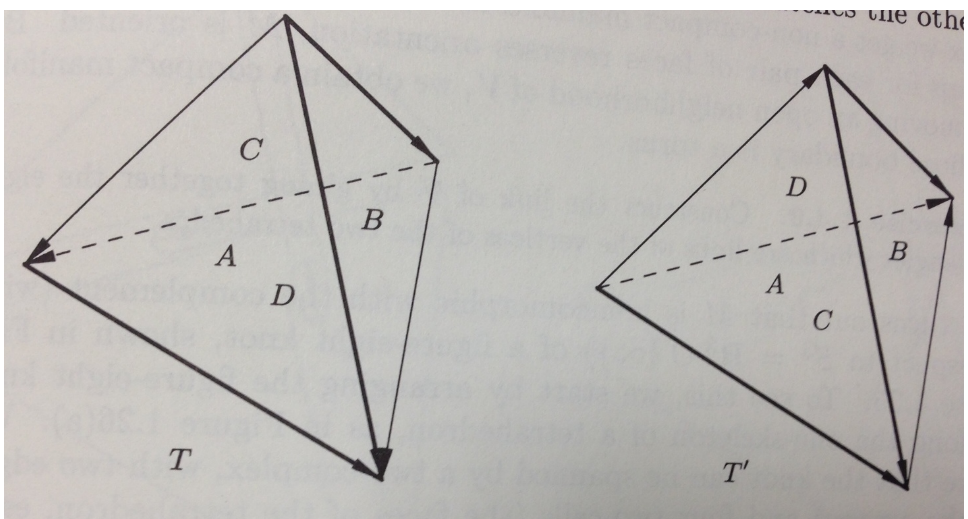

Figure 1.24. A simple three-dimensional gluing pattern. A simple pattern for gluing two tetrahedra. Each face has a label (centered on it), and faces with the same label are identified in a way that is unambiguously determined by the requirement that edge types (thick and thin) and directions match. (In fact, even the pairing of faces could be reconstructed from this requirement.)