Math 9023 Homework 2

Due Tuesday, 9/16/14

1. Prove that Seifert's algorithm, applied to the standard diagram of a (p, q) torus knot, yields a surface of genus $(p-1)(q-1)/2$.

2. Let S be a Seifert surface of genus g, constructed by applying Seifert's algorithm to a diagram. Prove that $\pi_1(S^3 \setminus S)$ is a free group of rank $2g$.

Note: Not all Seifert surfaces have complements with free fundamental group, so this shows that not every surface can come from Seifert's algorithm.

3. Let K be a (p, q, r) pretzel knot, where p, q, r are all odd multiples of 3. See Figure 1.7 in Lickorish's book for an image. Prove that the genus of K is exactly 1. *Hint:* What is the easiest way to show K is non-trivial?

4. A *two-bridge knot* $K \subset \mathbb{R}^3$ is one where the z–coordinate of the embedding function has exactly two local minima and two local maxima. Prove that a two-bridge knot is prime, by considering the intersections between a factorizing sphere and a sphere that separates the minima from the maxima.

5. Compute the Alexander polynomial of the figure-8 knot, using a spanning surface and the Seifert form.