Math 9023 Homework 1

Due Thursday, 9/4/14

1. Prove that the two-component links shown in Figure 1.15 of Prasolov–Sossinsky are all isotopic (and hence are all diagrams of the Whitehead link).

2. Show that any diagram of a knot K can be changed to a diagram of the unknot by changing some of the crossings from "over" to "under". How many crossing changes are necessary?

3. Let K be an oriented knot in oriented $\mathbb{R}^3$. Then the same knot with the opposite orientation is called the *inverse* of K, and denoted rK. If $f : \mathbb{R}^3 \to \mathbb{R}^3$ is an orientation–reversing homeomorphism (for instance, a reflection), then $f(K)$ is called the *mirror image* of K, and denoted $\overline{K}$.

Prove that the figure–8 knot is isotopic both to its inverse and to its mirror image.

4. A diagram of an oriented knot is shown on a screen using an overhead projector and a transparency. What knot $(K, rK, \overline{K}, \text{ or } r\overline{K})$ will appear on the screen if the transparency is turned over?

5. Do exercise 22(a) on page 55 of Hatcher, which verifies that the Wirtinger presentation is correct. Hatcher's book can be found online at

<center>http://www.math.cornell.edu/~hatcher/AT/ATch1.pdf</center>