Extra problem for Homework 5

Due Wednesday, 2/14/07

Problem 3 on Homework 3 implies that the manifold $\mathbb{RP}^3 \# \mathbb{RP}^3$ has fundamental group

$$\pi_1(\mathbb{RP}^3 \# \mathbb{RP}^3) \cong \mathbb{Z}_2 * \mathbb{Z}_2 \cong \langle a, b : a^2 = b^2 = 1 \rangle.$$

This group contains an index–2 subgroup

$$H = \{(ab)^n\} \cong \mathbb{Z}.$$

Prove that the double cover corresponding to this subgroup is $S^2 \times S^1$, by constructing an explicit covering map.