Math 8161 Homework 3

Due Thursday, 10/1/09

1. Find a metric space X and a subset $A \subset X$ that is closed and bounded, but not compact.

2. Let X be a Hausdorff space, and let A, B be disjoint compact subsets of X. Prove that there exist disjoint open sets U and V that contain A and B, respectively.

3. Find an open cover of $\mathbb{R}^2$ that does not have a Lebesgue number.

4. Prove or disprove the following:

a) The union of two compact subsets of X is compact.

b) The intersection of two compact subsets of X is compact.

5. Let X and Y be compact Hausdorff spaces, and $f : X \to Y$ a function. Let the *graph of f* be $G_f = \{(x, f(x)) : x \in X\}$. Prove or disprove the following statement: f is continuous if and only if G_f is a compact subset of $X \times Y$.

6. This problem needs to be corrected – see next homework.

Let X be the set of all lines in $\mathbb{R}^2$. For every line ℓ, and $\varepsilon > 0$, let $N_\varepsilon(\ell)$ be the set of all lines m that either intersect ℓ at angle less than ε, or lie parallel to ℓ at Euclidean distance less than ε.

a) Check that the collection $\{N_\varepsilon(\ell)\}$ forms a basis for a topology.

b) Let $Y \subset X$ be the set of all lines whose Euclidean distance to the origin is at most 1. Prove that Y is homeomorphic to a Möbius strip.