Math 8161 Homework 2

Due Tuesday, 9/22/09

1. Let A and B be subsets of a topological space X. Prove or disprove the following:

a) $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

b) $\overline{A \cap B} = \overline{A} \cap \overline{B}$.

2. Let X be a topological space, and let Δ be the *diagonal* of the product $X \times X$. That is, $\Delta = \{(x, x) : x \in X\}$. Prove that X is Hausdorff if and only if Δ is closed in $X \times X$.

3. Consider $\mathbb{R}$ with the standard topology, and let $\sim$ be the equivalence relation where $x \sim y$ iff $(x - y) \in \mathbb{Q}$. Prove that the quotient space $X = \mathbb{R}/\sim$ has the indiscrete topology.

4. Let X and Z be the following subspaces of $\mathbb{R}^2$ (with the standard topology):

$$X = \{(x, n) : n \in \mathbb{Z}\}, \qquad Z = \{(x, nx) : n \in \mathbb{Z}\}.$$

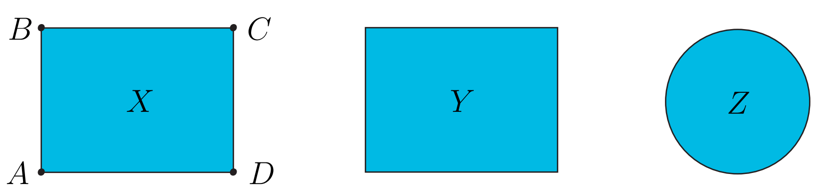
In other words, X is the union of horizontal lines at integer heights, and Z is the union of all lines through the origin with integer slope. From X we construct a quotient space $Y = X/\sim$, where $(0, n) \sim (0, m)$ for all m, n. Is Y homeomorphic to Z?

5. Let X, Y, and Z be the subsets of $\mathbb{R}^2$ drawn below. Describe, as precisely as possible, the following quotients:

a) $X/\sim$, where the segments AB and CD are identified to a single point.

b) $Y/\sim$, where the entire perimeter is identified to a single point.

c) $Z/\sim$, where every point on the circumference is identified to the opposite point on the circumference.



6. (Extra credit.) Consider $\mathbb{Z}$, with the topology generated by the bi-infinite sequences $S_{a,b} = \{a + nb : n \in \mathbb{Z}, b \neq 0\}$. Let $P \subset \mathbb{Z}$ be the set of prime numbers. Compute $\overline{P}$.