Math 8062 Homework 3

Due Wednesday, 2/4/26

1. Problem 16 on page 39 of Hatcher.

2. There is a standard way to glue together two (connected) manifolds M and N of the same dimension. Remove an open ball B^n from each of M and N, and glue $M \smallsetminus B^n$ to $N \smallsetminus B^n$ along the two $(n-1)$ dimensional boundary spheres. The resulting manifold is called the *connected sum* of M and N, and is denoted $M \# N$.

a) Prove that when $n \geq 3$, $\pi_1(M \smallsetminus B^n) \cong \pi_1(M)$.

b) Prove that when $n \geq 3$, $\pi_1(M \# N) \cong \pi_1(M) * \pi_1(N)$.