Math 8062 Homework 2

Due Wednesday, 1/28/26

1. Let X is be a path-connected space, and let $x_0, x_1 \in X$.

(a) Show that if g, h are paths from x_0 to x_1, then the change-of-basepoint isomorphisms $\beta_g, \beta_h \colon \pi_1(X, x_1) \to \pi_1(X, x_0)$ differ by an inner automorphism of $\pi_1(X, x_0)$. Conclude that if $\pi_1(X)$ is abelian, then there is a canonical isomorphism $\pi_1(X, x_1) \to \pi_1(X, x_0)$.

(b) Suppose that $\pi_1(X)$ is not abelian. What is an example of a nontrivial change-of-basepoint isomorphism $\pi_1(X, x_0) \to \pi_1(X, x_0)$? (Yes, both basepoints here are x_0.)

2, 3. Do problems 5, 8, 10 on page 38 of Hatcher.

4. Use the Ham Sandwich Theorem to solve the following problem in combinatorics.

Suppose there are 100 boxes, with each box containing some number of apples, oranges, and kiwis. Prove that there exists a way to choose 51 boxes out of 100, which will simultaneously contain at least half of the apples, at least half of the oranges, and at least half of the kiwis.

Remark: I am not aware of any solution that doesn't use topology. However, I will certainly accept it if you find one! If the problem is simplified to 2 kinds of fruit instead of 3, then there is a combinatorial solution by induction (although it still requires a bit of cleverness).