Math 8062 Homework 10

Due Thursday, 4/21/22

1. Let M and N be closed, connected, oriented manifolds of dimension n. Let $M\#N$ be their connected sum. Prove that

$$\tilde{H}_k(M\#N) \;\cong\; \tilde{H}_k(M) \oplus \tilde{H}_k(N),$$

for all $k \neq n$. (*Hint:* be careful when $k = n - 1$!)

2. Page 155, problem 2.

3. Let M be a compact, connected, orientable 4–manifold, such that $\pi_1(M) \cong A_5$. Prove that every vector field on M must have a zero.

Hint: You may take as given that $H_1(M) \cong H_3(M)$, a special case of Poincaré duality.

4. Let S_g and S_h be surfaces of genus g and h, respectively. Suppose $g > h$.

a) Construct a map $f : S_g \to S_h$ of degree 1.

b) Prove that every map $\varphi : S_h \to S_g$ has degree 0. *Hint*: Consider the rank of $\pi_1(S_g)$ and $\pi_1(S_h)$, where rank is the minimum number of elements required to generate the group. If $deg(\varphi) > 0$, results from class give some information that should lead to a contradiction.