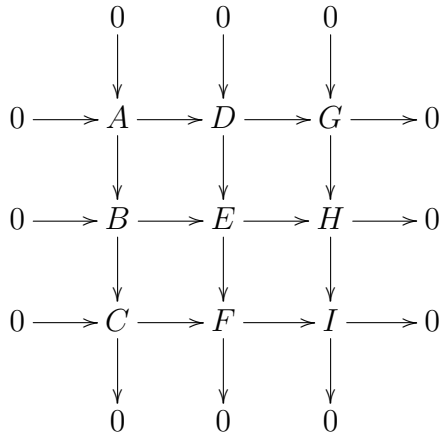
Math 8062 Homework 9

Due Thursday, 4/11/18

1. Consider the following commutative diagram of Abelian groups:



Prove that if all the rows are exact, and the second and third columns are exact, then the first column is also exact. *Note:* When describing a diagram chase, it is helpful to give "walking directions." For instance: given an element $e \in E$, chase down, then to the right, then up...

2. Do problem 29 on page 158 of Hatcher.

3. Let X be the same 3–manifold as in the previous problem (page 158, #29). Show that X has a non-vanishing vector field.

4. Let M and N be closed, connected, oriented manifolds of dimension n. Let $M\#N$ be their connected sum. Prove that

$$\tilde{H}_k(M\#N) \;\cong\; \tilde{H}_k(M) \oplus \tilde{H}_k(N),$$

for all $k \neq n$. *Hint: be careful when $k = n - 1$!*