Math 8062 Homework 3

Due Thursday, 2/14/19

1. There is a standard way to glue together two (connected) manifolds M and N of the same dimension. Remove an open ball B^n from each of M and N, and glue $M \smallsetminus B^n$ to $N \smallsetminus B^n$ along the two $(n-1)$ dimensional boundary spheres. The resulting manifold is called the *connected sum* of M and N, and is denoted $M \# N$.

a) Prove that when $n \geq 3$, $\pi_1(M \smallsetminus B^n) \cong \pi_1(M)$.

b) Prove that when $n \geq 3$, $\pi_1(M \# N) \cong \pi_1(M) * \pi_1(N)$.

2. Let M be an orientable manifold of dimension $n \geq 4$. Let C be a knot (i.e., an embedded circle) in M. Let $V \cong B^{n-1} \times S^1$ be an open neighborhood of C. (Here, B^{n-1} is an open ball of dimension $n-1$.)

a) Prove that $\pi_1(M \smallsetminus V) \cong \pi_1(M)$. *Hint:* Put a cell complex structure on $\overline{V}$, and attach these cells one at a time.

b) Is the hypothesis that $n \geq 4$ necessary?

3. Problem 8 on page 53 of Hatcher.

4. Problem 14 on page 54 of Hatcher. For extra credit, show that X is a 3–manifold.

General hint: for some of these problems, van Kampen's theorem will be more useful than Proposition 1.26 (the application to cell complexes).