Math 8062 Homework 10

Due Thursday, 4/18/19

1. Page 155, problem 2.

2. Let M be a compact, connected, orientable 4–manifold, such that $\pi_1(M) \cong A_5$. Prove that every vector field on M must have a zero.

Hint: You may take as given that $H_1(M) \cong H_3(M)$, a special case of Poincaré duality.

3. Let M^n and N^n be smooth n–manifolds, where M is orientable and N is not.

a) Let $f : M \to N$ be a smooth map, and assume f is an immersion (i.e. Df has full rank at every point). Prove that f lifts to the orientation double cover of N.

b) Find an example to demonstrate that the immersion hypothesis is necessary.

4. Let S_g and S_h be surfaces of genus g and h, respectively. Suppose $g > h$.

a) Construct a map $f : S_g \to S_h$ of degree 1.

b) Prove that every map $\varphi : S_h \to S_g$ has degree 0. *Hint*: Consider the rank of $\pi_1(S_g)$ and $\pi_1(S_h)$, where rank is the minimum number of elements required to generate the group. If $deg(\varphi) > 0$, results from class give some information that should lead to a contradiction.