

## Math 8062 Homework 9

Due Wednesday, 4/18/11

1. Page 155, problem 2.
2. Page 155, problem 3.
3. Let  $M^n$  and  $N^n$  be smooth  $n$ -manifolds, where  $M$  is orientable and  $N$  is not.
  - a) Let  $f : M \rightarrow N$  be a smooth map, and assume  $f$  is an immersion (i.e.  $Df$  has full rank at every point). Prove that  $f$  lifts to the orientation double cover of  $N$ .
  - b) Find an example to demonstrate that the immersion hypothesis is necessary.
4. Let  $S_g$  and  $S_h$  be surfaces of genus  $g$  and  $h$ , respectively. Suppose  $g > h$ .
  - a) Construct a map  $f : S_g \rightarrow S_h$  of degree 1.
  - b) Prove that every map  $\varphi : S_h \rightarrow S_g$  has degree 0. *Hint:* Consider the rank of  $\pi_1(S_g)$  and  $\pi_1(S_h)$ , where rank is the minimum number of elements required to generate the group. If  $\deg(\varphi) > 0$ , results from class give some information that should lead to a contradiction.