

## Math 8062 Homework 6

Due Wednesday, 3/19/11

**1, 2, 3.** Do Problems 5, 8, 10 on Page 131 of Hatcher.

**4.** Problem 1 of the third homework implies that the manifold  $\mathbb{RP}^3 \# \mathbb{RP}^3$  has fundamental group

$$\pi_1(\mathbb{RP}^3 \# \mathbb{RP}^3) \cong \mathbb{Z}/2 * \mathbb{Z}/2 \cong \langle a, b : a^2 = b^2 = 1 \rangle.$$

This group contains an index-2 subgroup  $H = \{(ab)^n\} \cong \mathbb{Z}$ . Prove that the double cover corresponding to this subgroup is  $S^2 \times S^1$ , by constructing an explicit covering map. *Hint:* to do this, it helps to construct a fundamental domain for  $\mathbb{RP}^3 \# \mathbb{RP}^3$ .