Math 8062 Homework 7

Due Wednesday, 4/17/11

1. Do problem 29 on page 158 of Hatcher.

2. Let M and N be closed, connected, oriented manifolds of dimension n. Let $M\#N$ be their connected sum. Prove that

$$\tilde{H}_k(M\#N) \cong \tilde{H}_k(M) \oplus \tilde{H}_k(N),$$

for all $k \neq n$. *Hint: be careful when* $k = n - 1$*!*

3. Sort out the diagram–chasing proof of problem 38 on page 159 of Hatcher, and be ready to explain it orally. Observe that this gives an axiomatic proof of the Mayer-Vietoris sequence.

4. Let M^n and N^n be smooth n–manifolds, where M is orientable and N is not.

a) Let $f : M \to N$ be a smooth map, and assume f is an immersion (i.e. Df has full rank at every point). Prove that f lifts to the orientation double cover of N.

b) Find an example to demonstrate that the immersion hypothesis is necessary.

5. [Extra credit] Let S_g and S_h be surfaces of genus g and h, respectively. Suppose $g > h$.

a) Construct a map $f : S_g \to S_h$ of degree 1.

b) Prove that every map from S_h to S_g has degree 0. *Hint: consider the rank of* $\pi_1(S_g)$ *and* $\pi_1(S_h)$*, where rank is the minimum number of elements required to generate the group.*