Math 8062 Homework 3

Due Wednesday, 2/27/11

1. There is a standard way to glue together two (connected) manifolds M and N of the same dimension. Remove an open ball B^n from each of M and N, and glue $M \setminus B^n$ to $N \setminus B^n$ along the two $(n-1)$ dimensional boundary spheres. The resulting manifold is called the *connected sum* of M and N, and is denoted $M \# N$.

a) Prove that when $n \geq 3$, $\pi_1(M \setminus B^n) \cong \pi_1(M)$.

b) Prove that when $n \geq 3$, $\pi_1(M \# N) \cong \pi_1(M) * \pi_1(N)$.

2. Let M be a manifold of dimension $n \geq 4$, and let C be a knot (i.e., an embedded circle) in M.

a) Prove that $\pi_1(M \setminus C) \cong \pi_1(M)$.

b) Is the hypothesis that $n \geq 4$ necessary?

3. Let S be a compact, connected surface with boundary. Let $X = X^2$ be a cell complex structure on S, in which every 2–cell is an embedded polygon. Prove that X deformation retracts into the 1–skeleton X^1, and conclude that $\pi_1(S)$ is a free group.

For extra credit, extend this argument to dimension n: that is, show an n–manifold with boundary deformation retracts into its $(n-1)$ skeleton.

4. Do problem 8 on page 53 of Hatcher.

5. Do problem 14 on page 54 of Hatcher.

General hint: for some of these problems, van Kampen's theorem will be more useful than Proposition 1.26 (the application to cell complexes).