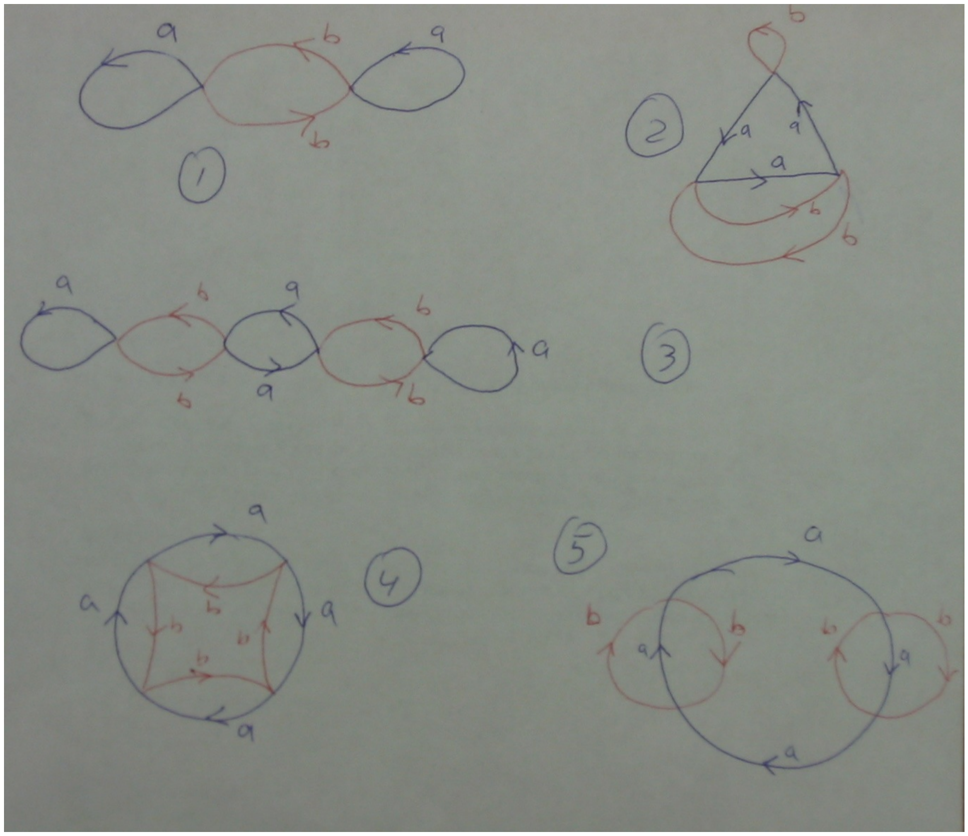
Math 8062 Homework 5

Due Thursday, 3/3/11

1. Which of the following covering spaces of $S^1 \vee S^1$ are regular, i.e. correspond to normal subgroups?



2. Let X be the cell complex obtained by identifying opposite faces of a cube with a $90°$ twist (as in problem 14 on page 54). Prove that every map $f : X \to S^1$ is homotopic to a constant map. *Hint:* think about lifting maps to covering spaces.

3. Problem 1 of the third homework implies that the manifold $\mathbb{RP}^3 \# \mathbb{RP}^3$ has fundamental group

$$\pi_1(\mathbb{RP}^3 \# \mathbb{RP}^3) \cong \mathbb{Z}/2 * \mathbb{Z}/2 \cong \langle a, b : a^2 = b^2 = 1 \rangle.$$

This group contains an index–2 subgroup $H = \{(ab)^n\} \cong \mathbb{Z}$. Prove that the double cover corresponding to this subgroup is $S^2 \times S^1$, by constructing an explicit covering map.

4. Do problem 18 on page 80 of Hatcher.

5. Let X be the space obtained from a torus $S^1 \times S^1$ by attaching a Möbius band via a homeomorphism from the boundary circle of the Möbius band to the circle $S^1 \times \{x_0\}$ in the torus. Compute $\pi_1(X)$, describe the universal cover $\widetilde{X}$ of X, and describe the action of $\pi_1(X)$ on $\widetilde{X}$. *Hint:* first, construct a double cover.