Math 8061 Homework 5

Due Wednesday, 10/11/23

1. Problem 5-1 in Lee.

2. Problem 5-6 in Lee.

3. Let C be a circle smoothly embedded in $\mathbb{R}^4$. The goal of this problem is to use Sard's theorem to show that there exists a 3–dimensional hyperplane H, such that the orthogonal projection $\pi : C \to H$ is an embedding.

Let $v \in S^3$ be a unit vector in $\mathbb{R}^4$. Perpendicular to v is a hyperplane $H_v \cong \mathbb{R}^3$. Let $\pi_v : \mathbb{R}^4 \to H_v$ be the orthogonal projection onto this hyperplane. Then we also obtain a smooth map $\pi_v : C \to H_v$

a) Construct a smooth map $f : X \to S^3$, for some suitably constructed manifold X, such that $v \in S^3$ is a critical value of f precisely when $\pi_v : C \to H_v$ fails to be an immersion.

b) Construct a smooth map $g : Y \to S^3$, for some suitably constructed manifold Y, such that $v \in S^3$ is a critical value of g precisely when $\pi_v : C \to H_v$ fails to be 1–1.

c) Now, use Sard's theorem to conclude that almost every $v \in S^3$ is a regular value of both f and g (that is, a critical value of neither f nor g), hence the corresponding projection π_v is an embedding.