Math 8061, Homework 10

Due Friday 12/8/23

1. Problem 14–6 in Lee.

2. Suppose that ω is a closed form on M, and η is exact. Prove that $\omega \wedge \eta$ is exact.

3. Suppose M and N are connected, oriented smooth n-manifolds, and $f: M \to N$ is an immersion. Prove that f is orientation-preserving everywhere or orientation-reversing everywhere.

4. Problems on orientations:

a) If M is orientable, prove that an open set $U \subset M$ is orientable.

b) If M and N are orientable, prove that $M \times N$ is orientable. (Compare Exercise 15.8)

c) If M is not orientable, prove that $M \times N$ is not orientable for any N. *Hint:* Use part (a) and induction on dimension.