Math 8061 Homework 9

Due Wednesday, 11/18/21

1. Do problem 14–1 of Lee.

2. A k–form η on a vector space V is called *decomposable* if it can be expressed as

$$\eta = \omega^1 \wedge \cdots \wedge \omega^k,$$

for 1–forms $\omega^1, \ldots, \omega^k$.

a) Prove that every 2–form on $\mathbb{R}^3$ is decomposable.

b) On $V = \mathbb{R}^4$, with the standard basis $\epsilon^1, \ldots, \epsilon^4$ for V^*, consider the 2–form $\eta = \epsilon^1 \wedge \epsilon^2 + \epsilon^3 \wedge \epsilon^4$. Show that $\eta \wedge \eta \neq 0$, and then prove that this implies η is not decomposable.

3. Do problem 14–6 of Lee.