Math 8061 Homework 7

Due Wednesday, 11/4/21

1. Compute the flow of each of the following vector fields on $\mathbb{R}^2$:

a) $X = y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}$

b) $Y = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}$

c) $Z = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}$

d) $W = x\frac{\partial}{\partial x} - 2y\frac{\partial}{\partial y}$

2. Let the vector fields X, Y, Z be as in the last problem.

a) Compute that $[X, Y] \neq 0$, and that their flows do not commute.

b) Compute that $[Y, Z] = 0$, and that their flows commute.

3. Problem 9–2 in Lee.

4. Problem 9–4 in Lee.

5. (Ungraded exercise) Let $f : M \to N$ be a diffeomorphism, and let X and Y be vector fields on M. Check that $f_*[X, Y] = [f_*X, f_*Y]$.

You can find a proof of this in many places, including Lie's book. But it's a nice exercise in unwinding the definitions to sort this out for yourself, so I recommend doing that.