Math 8061 Homework 4

Due Thursday, 9/30/21

1. Let M be a smooth compact n–manifold. Prove that there is no submersion $f : M \to \mathbb{R}^k$, for any $k > 0$. *Hint:* think of a function that would have to achieve a maximum value on $f(M)$.

2. If M is compact, prove that every 1–1 immersion $f : M \to N$ is an embedding.

3. Problem 5–7 in Lee's book.

4. Let $\mathbb{RP}^2 = S^2/\sim$, where antipodal points on the sphere are identified.

a) Prove that the map $g : \mathbb{RP}^2 \to \mathbb{R}^4$, defined by

$$g([x, y, z]) = (yz,\ xz,\ xy,\ x^2 - y^2),$$

is a smooth embedding.

b) Prove that the map $g : \mathbb{RP}^2 \to \mathbb{R}^3$, defined by

$$g([x, y, z]) = (yz,\ xz,\ xy)$$

fails to be an immersion at 6 points (the 6 image points are at distance 1/2 from the origin, on each axis). In fact, there is an immersion $h : \mathbb{RP}^2 \to \mathbb{R}^3$, called Boy's surface. See

`http://en.wikipedia.org/wiki/Boy's_surface`